Uncollapsing, Decoherence Suppression, and Quantum Error Correction/Detection with Phase Qubits

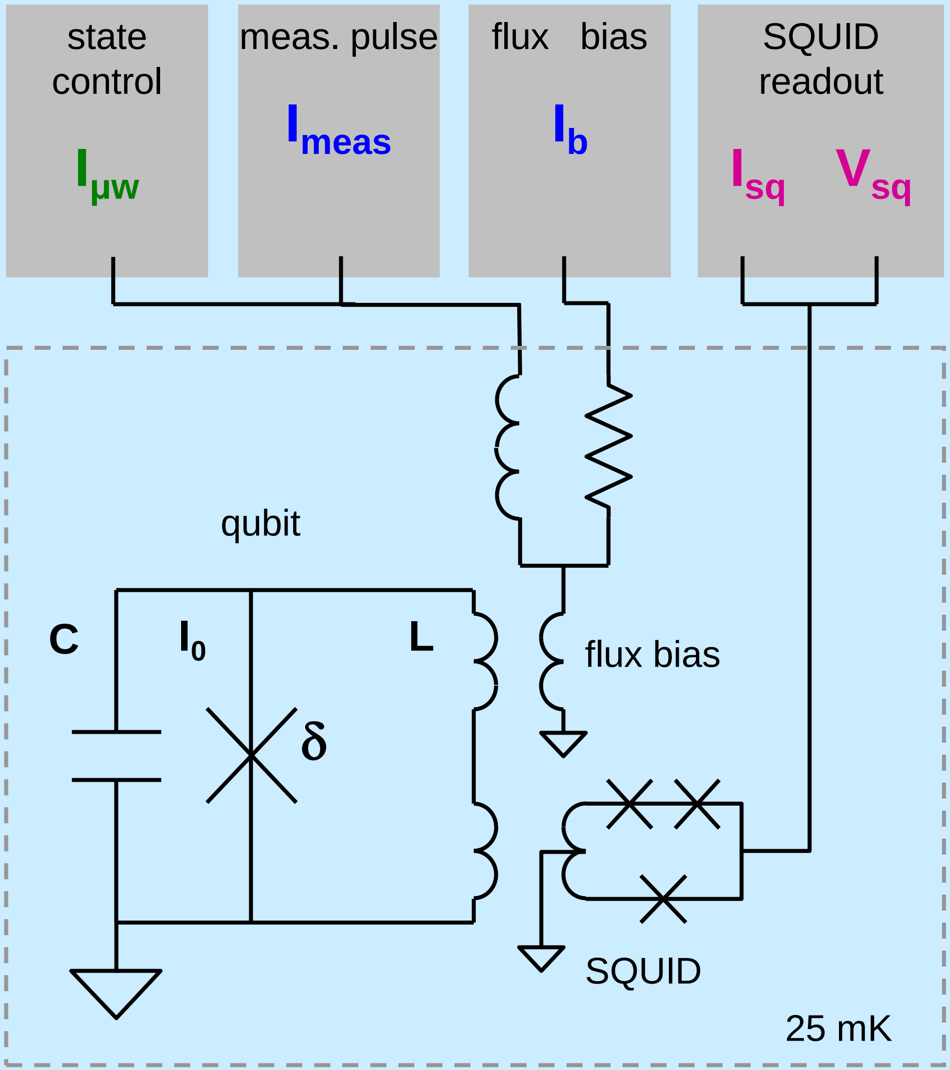
Oral Qualifying Examination

Kyle Keane

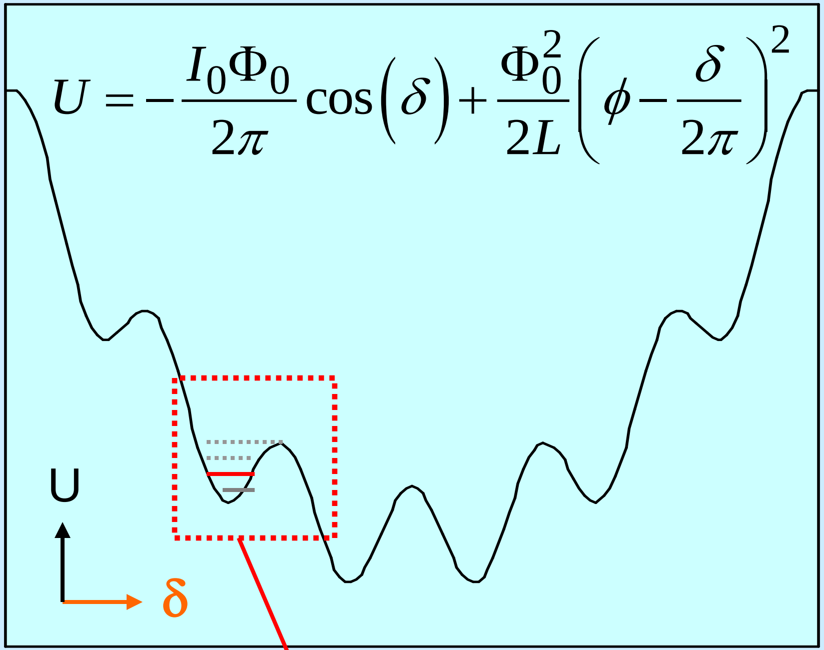
Outline
-Introduction to phase qubit and partial measurement (5 slides)
-Explanation of experimental results of uncollapsing (8 slides)
-Proposed experiment for suppressing decoherence using uncollapsing (3 slides)
-Redundant coding to protect against x rotations (6 slides)
-Performance of these codes for detection of relaxation errors (4 slides)
-Two qubit quantum error detection of rotations in presence of dephasing (2 slides)
-Future directions (1 slide)

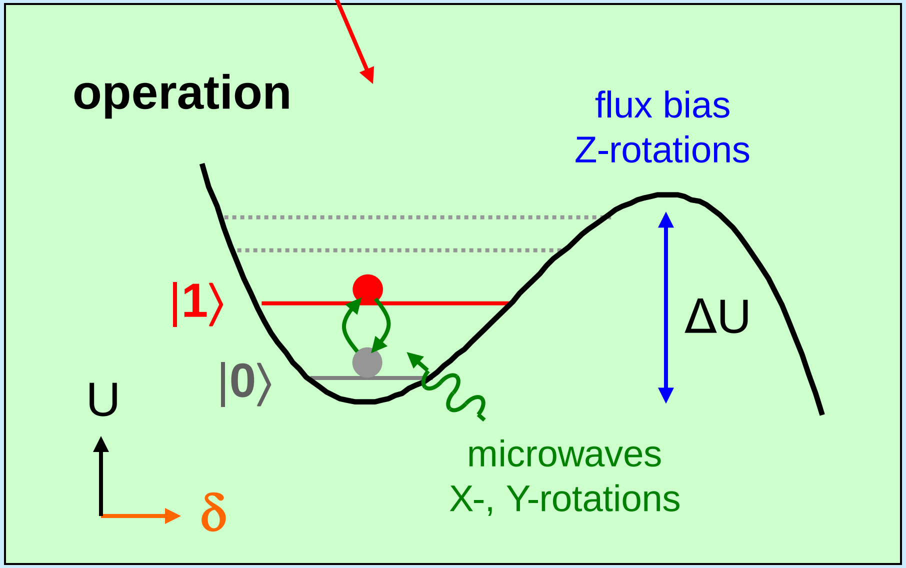
Flux Biased Phase Qubit

phase qubit circuit



$$U = -\frac{I_0\Phi_0}{2\pi}\cos(\delta) + \frac{\Phi_0^2}{2L}\left(\phi - \frac{\delta}{2\pi}\right)^2$$



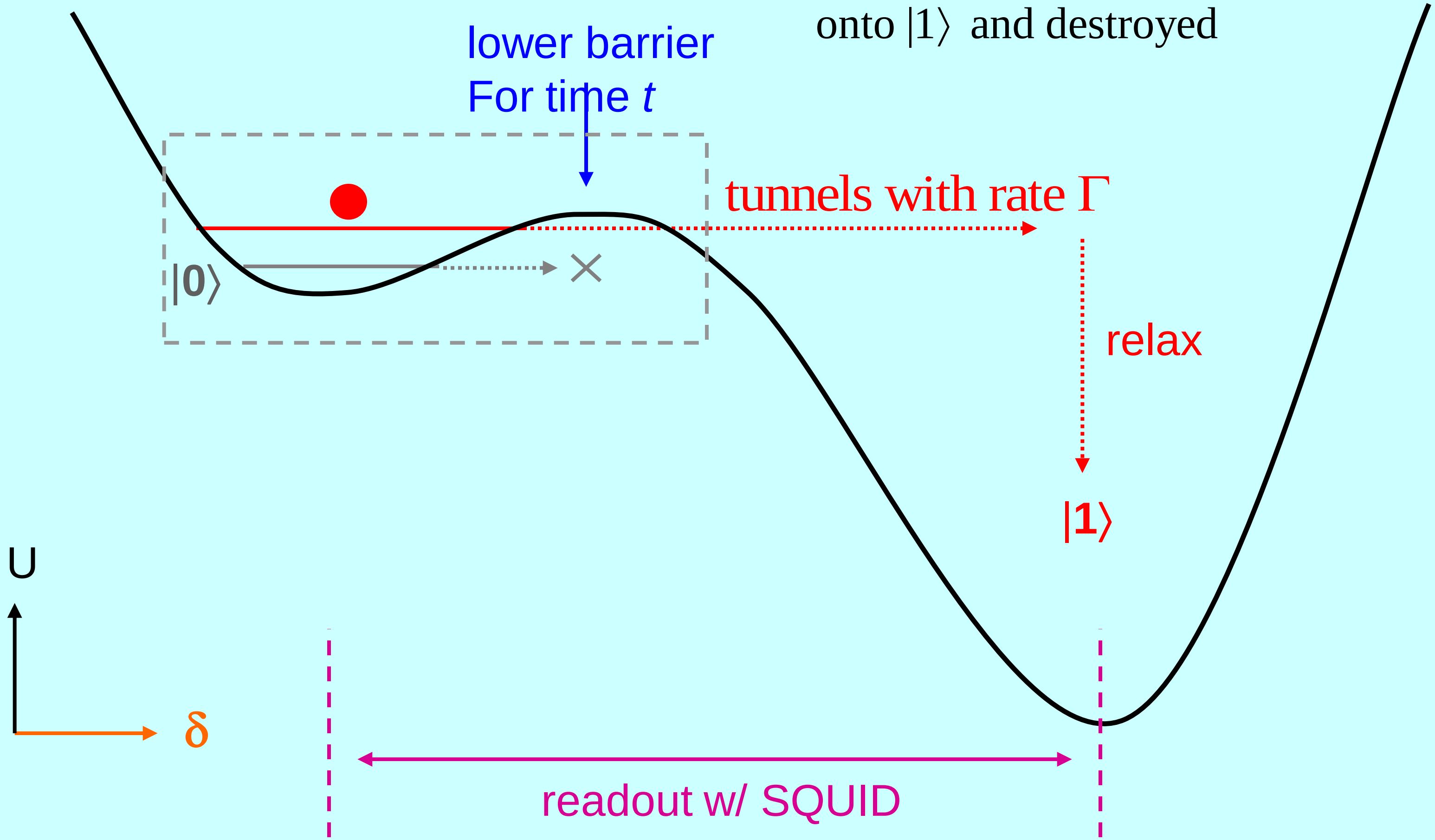


Full Measurement Using Tunneling

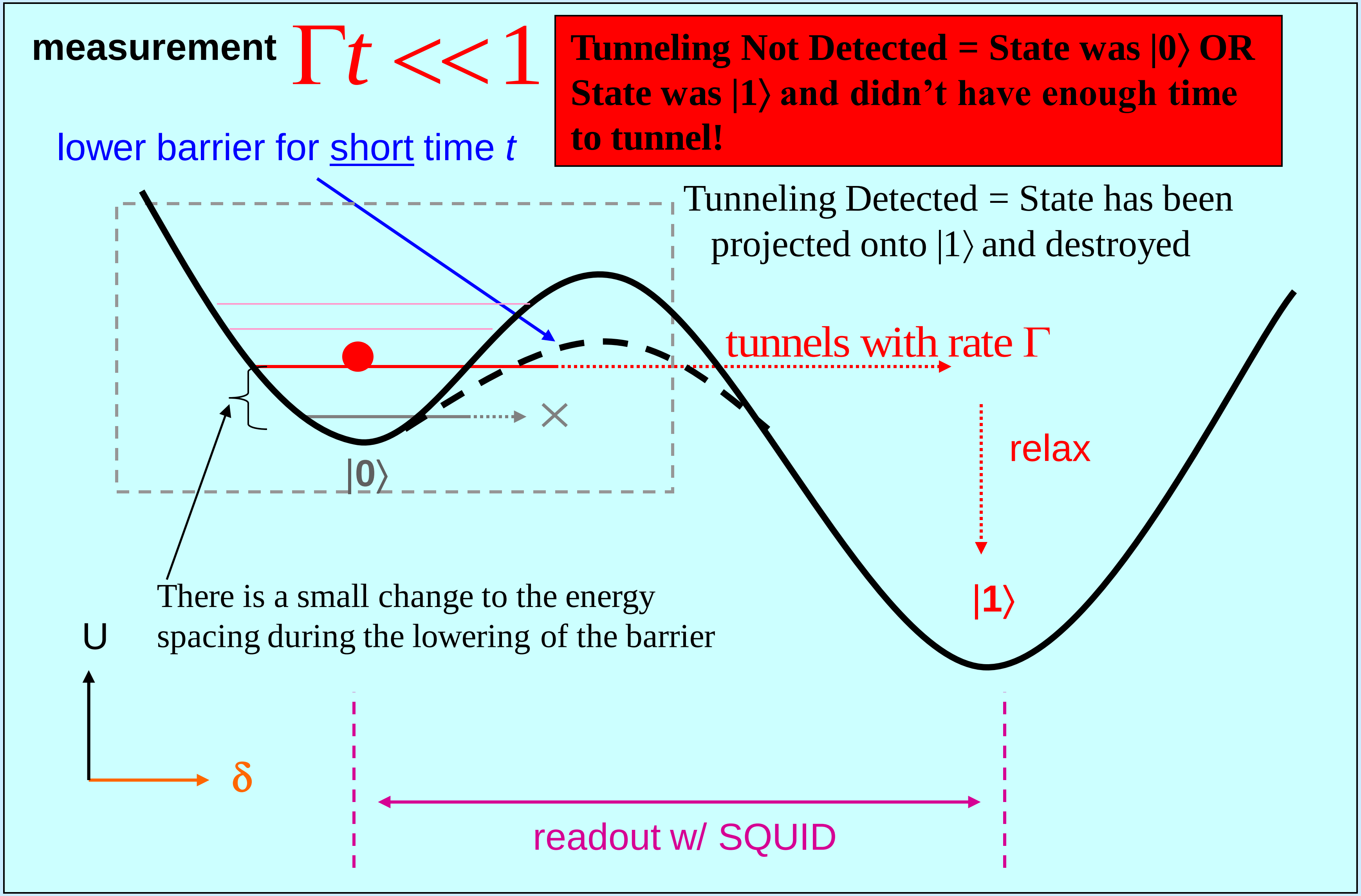
measurement $\Gamma t \approx 1$

Tunneling Not Detected = State has been projected onto $|0\rangle$

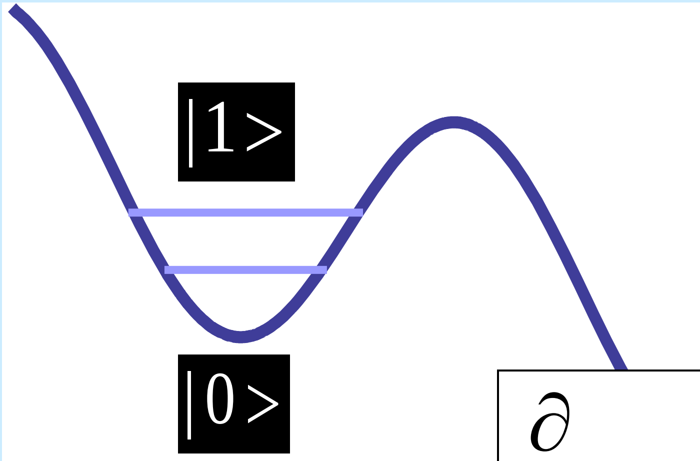
Tunneling Detected = State has been projected onto $|1\rangle$ and destroyed

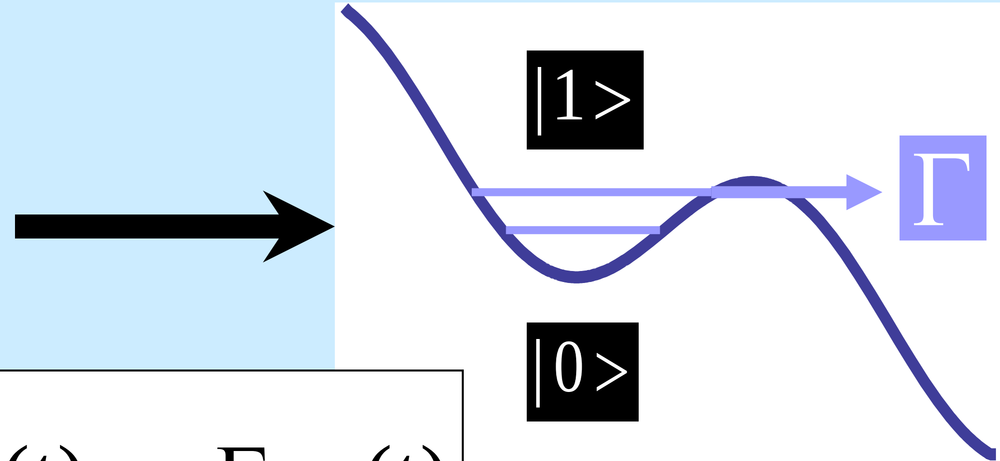


Partial measurement



Bayesian Description of State Evolution





$$\frac{\partial}{\partial t}\rho_{11}(t) = -\Gamma\rho_{11}(t)$$

$$\rho_{11}(t) = \rho_{11}(0)e^{-\Gamma t}$$

$$\rho_{00}(t) = \rho_{00}(0)$$

$$e^{-\Gamma t} \equiv 1 - p(t)$$

$$\rho_{01}(t) = \rho_{01}(0)\sqrt{1-p(t)}$$

$$\sqrt{\rho_{00}(t)\rho_{11}(t)} = e^{i\varphi}\sqrt{\rho_{00}(0)\rho_{11}(0)}$$

$$\alpha|0\rangle + \beta|1\rangle \to \frac{\alpha|0\rangle + \beta e^{i\varphi}\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}$$

Process Fidelities

How close to ideal is a process	What does a process do

How close to ideal is a process

operator sum decomposition
for a quantum operation £

$$£(\rho) = \sum_j A_j \rho A_j^\dagger$$

choose a complete set of operators $\{A_m\}$

$$A_j = \sum_m a_{j,m} A_m$$

$$£(\rho) = \sum_j \sum_{m,n} a_{j,m} A_j \rho \, a^*_{j,n} A_j^\dagger$$

Process Matrix

$$\chi_{m,n} \equiv \sum_j a_{j,m} a^*_{j,n}$$

$$£(\rho) = \sum_{m,n} \chi_{m,n} A_j \rho A_j^\dagger$$

Chi Fidelity

$$F_\chi = Tr(\chi_{ideal} \chi_{real})$$

What does a process do

Average Fidelity

$$\overline{F} = \int <\psi_{in} | £(\rho) | \psi_{in}> d|\psi_{in}>$$

On average

how far does £ bring ρ

from its initial state

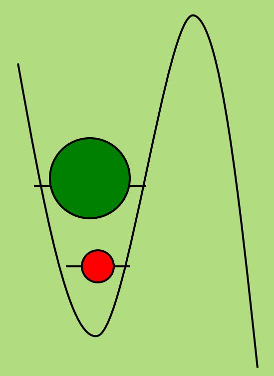
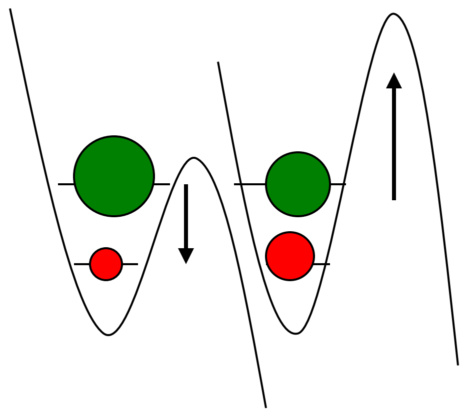
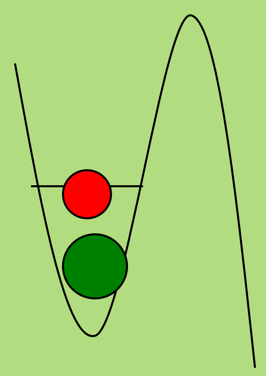
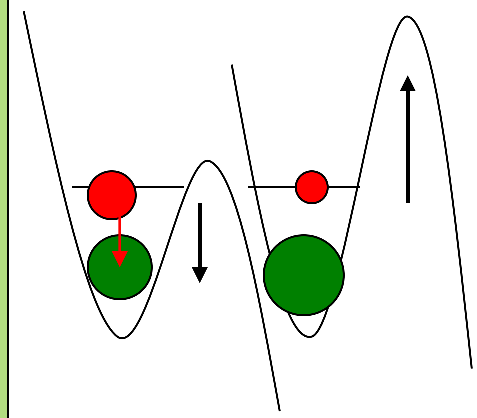
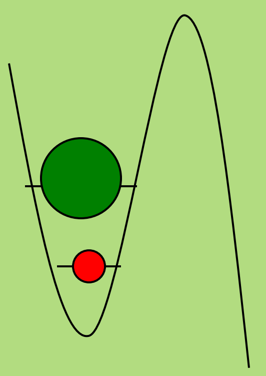
Relation

$$\overline{F} = \frac{d\, F_\chi + 1}{d + 1}$$

What do we need?

$$£(\rho), \rho$$

Project 1: Uncollapsing Experiment

State Prepared	Partial Measurement Projects state toward 0	π rotation	Partial Measurement Projects state toward 0	π rotation
				
	Doesn’t Tunnel		Doesn’t Tunnel	

If tunneling does not occur, the qubit state is recovered

In experiment, only data for cases where tunneling does not occur is kept

Ideal Theory

Partial Measurement
No tunneling,
Bayesian update

Pi rotation

Partial Measurement
No tunneling,
Bayesian update

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \frac{\alpha|0\rangle+\beta e^{\varphi}\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \rightarrow \frac{\beta e^{\varphi}\sqrt{1-p}\,|0\rangle+\alpha|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \rightarrow \frac{\beta e^{\varphi}\sqrt{1-p}\,|0\rangle+\alpha e^{\varphi}\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2(1-p)+|\beta|^2(1-p)}} = \beta|0\rangle+\alpha|1\rangle$$

At each partial measurement there is a probability that the qubit will tunnel. Therefore, there is a probability that this procedure will destroy the qubit, otherwise you have performed a Pi rotation.

$$F_{\chi}(perfect\ \pi\ rotation)=1$$

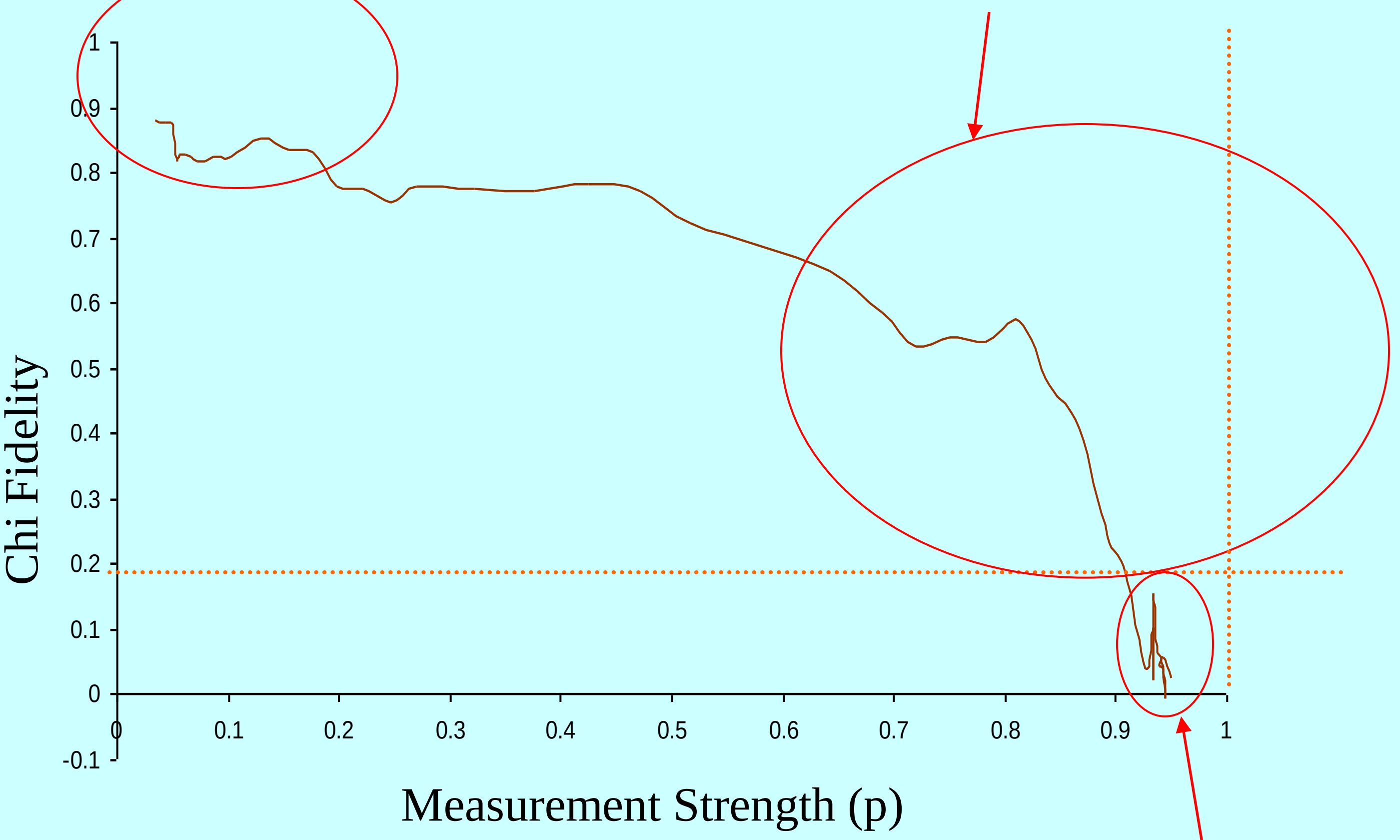
The fidelity should be independent of the measurement strength!

$$F_{\chi}(process)=1 \qquad 0\le p<1$$

Questions of Theoretical Interest

Why does it not start from 1?

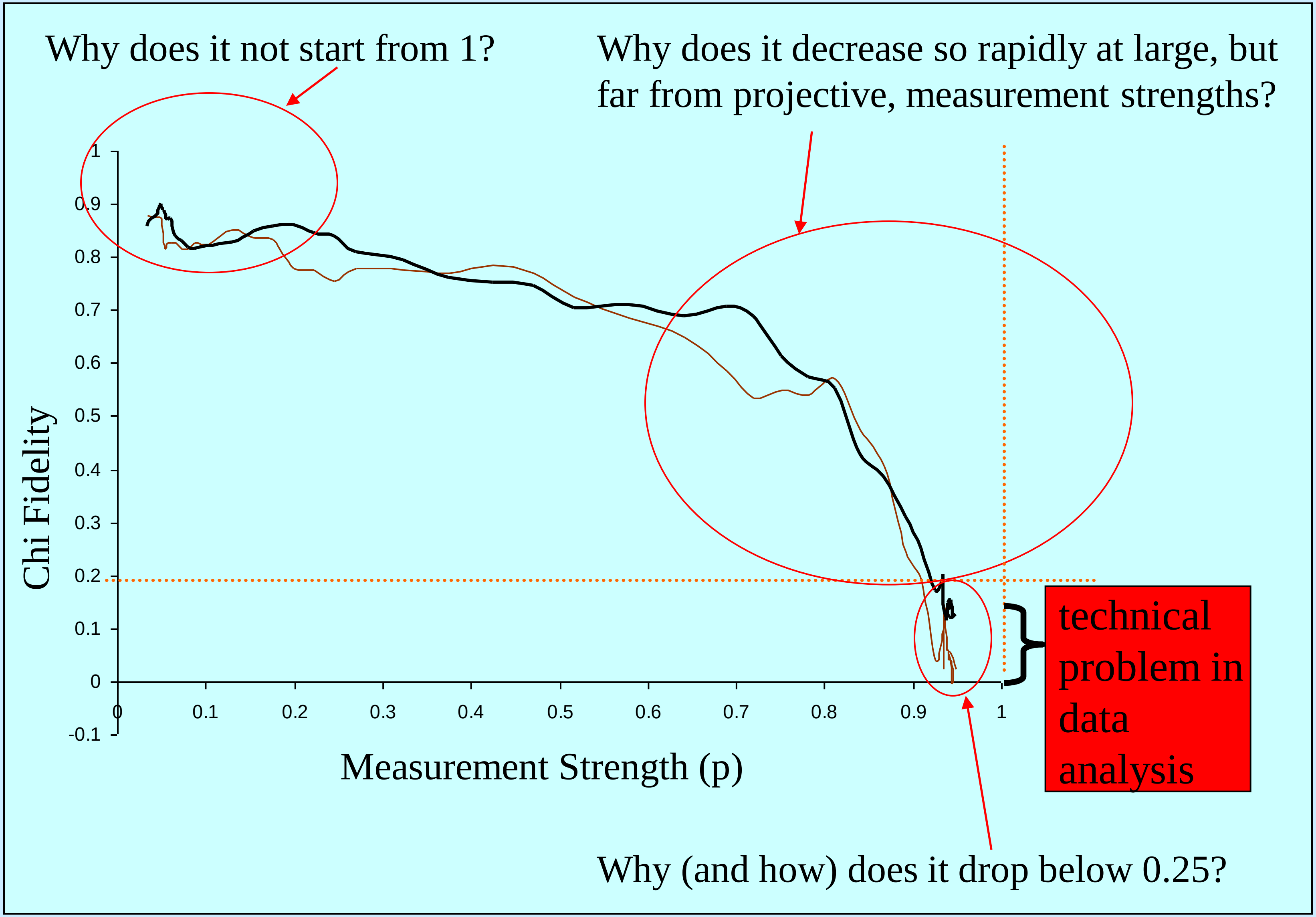
Why does it decrease so rapidly at large, but far from projective, measurement strengths?



Why (and how) does it drop below 0.25?

N. Katz et al., Phys. Rev. Lett. 101, 200401 (2008)

Understanding Their Data Analysis



N. Katz et al., Phys. Rev. Lett. 101, 200401 (2008)

Simple Analytics-Just Relaxation

No Relaxation

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \frac{\alpha|0\rangle+\beta e^{\varphi}\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \rightarrow \frac{\beta e^{\varphi}\sqrt{1-p}\,|0\rangle+\alpha|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \rightarrow \frac{\beta e^{\varphi}\sqrt{1-p}\,|0\rangle+\alpha e^{\varphi}\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2(1-p)+|\beta|^2(1-p)}} = \beta|0\rangle+\alpha|1\rangle$$

Relaxation Possible | Relaxation Possible | Relaxation Possible | Relaxation Possible

With Relaxation-

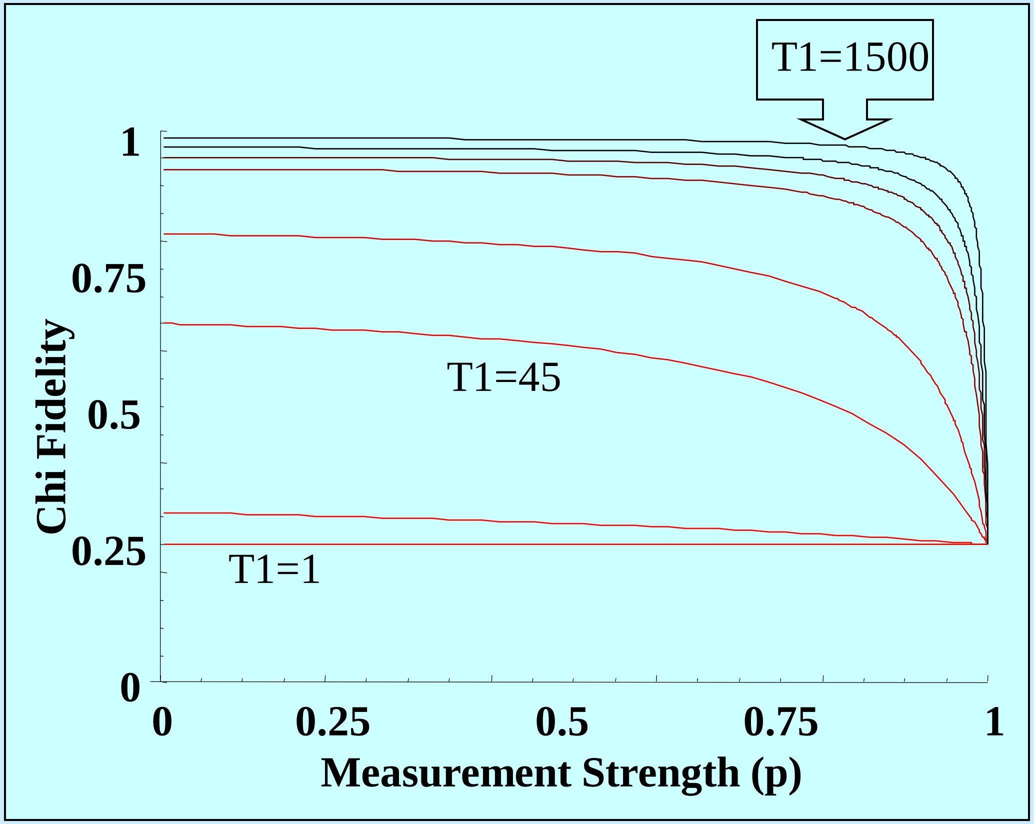
We unravel the continuous process of relaxation into discrete outcomes with probabilities

Similar to treatment of partial measurement

$$\alpha|0\rangle+\beta|1\rangle \rightarrow \begin{cases} \dfrac{\alpha|0\rangle+\beta e^{-t/2T_1}|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2 e^{-t/T_1}}} & \text{with probability } |\alpha|^2+|\beta|^2 e^{-t/T_1} \\ |0\rangle & \text{with probability } |\beta|^2\left(1-e^{-t/T_1}\right) \end{cases}$$

Important Results From Analytics

The Effect of Relaxation (T1) on Fidelity



Duration of Process = **44** ns

T1(ns)= 1, 10, 45, 100, 300, **450**, 700, 1500

F(p=0)

$$\frac{1}{4}\left(1+e^{-t/T_1}+2e^{-t/T_2}\right)$$

Duration of Process = t

Universal Scaling of F(p near 1)

$$F_\chi = \frac{1}{4}\left(1+\frac{1}{1+\frac{1}{x}}+\frac{2}{1+\frac{1}{2x}}\right)$$

where $x = \frac{1-p}{1-e^{-t_3/T_1}}$

t_3 = the amount of time between the Pi rotation and the second measurement

Numerics-What Else *Might Be* Happening

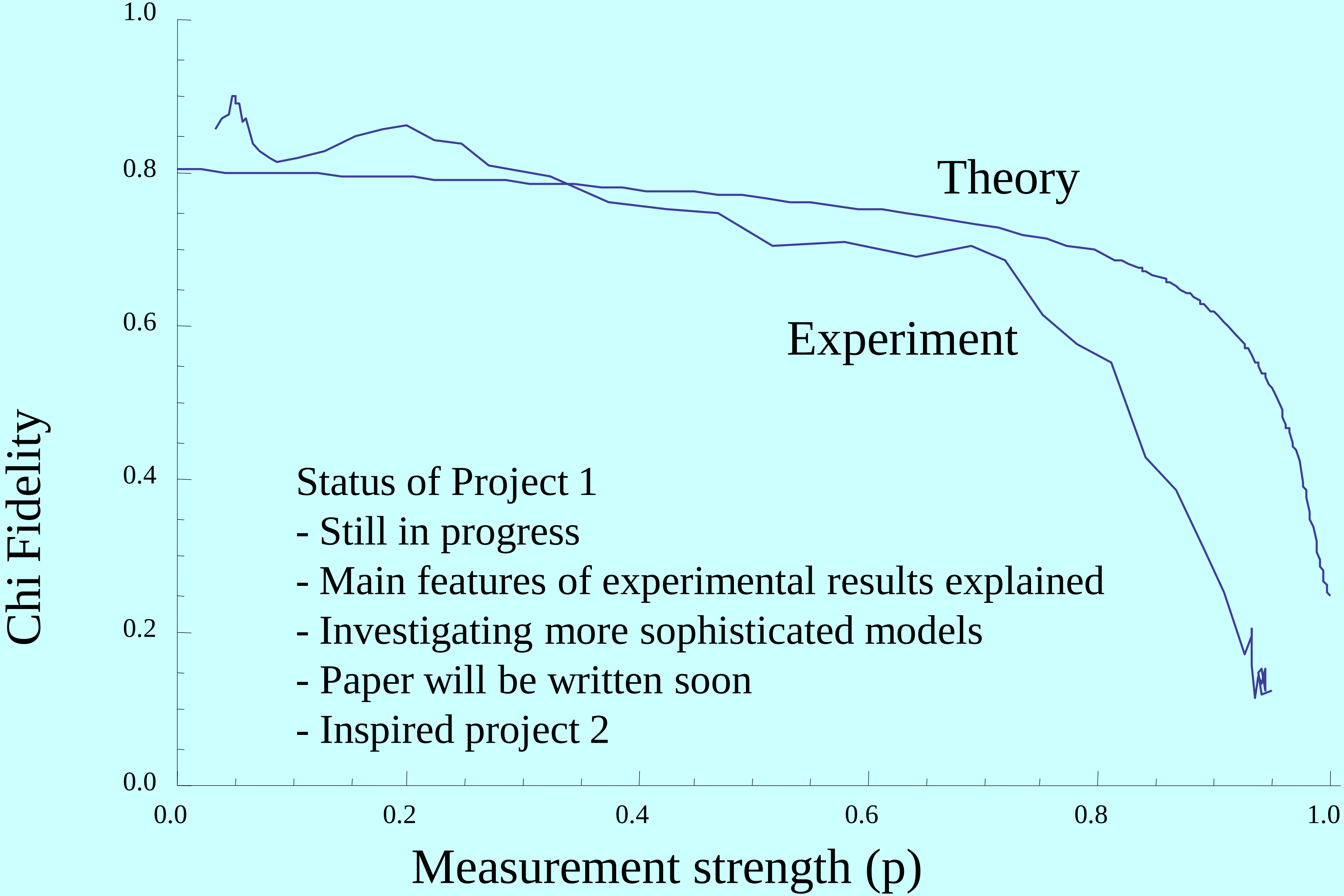
LINK

Relaxation and Dephasing Throughout

Experimental Protocol

-state prep (3 ns) →	Assumptions -Known pure state was prepared -Higher levels have not been populated
-partial measurement (4 ns) → -wait (3 ns)	-additional dephasing -spurious excitations -higher levels tunnel -measurement fidelity
-Pi rotation (10 ns) → -wait (3 ns)	-rotation angle skewed -damping of Rabi oscillations
-partial measurement (4 ns) →	-same as first -change in level splitting
-tomography (10 ns) →	-additional decoherence -asymmetry of measuring x, y, and z -higher levels tunnel -measurement fidelity

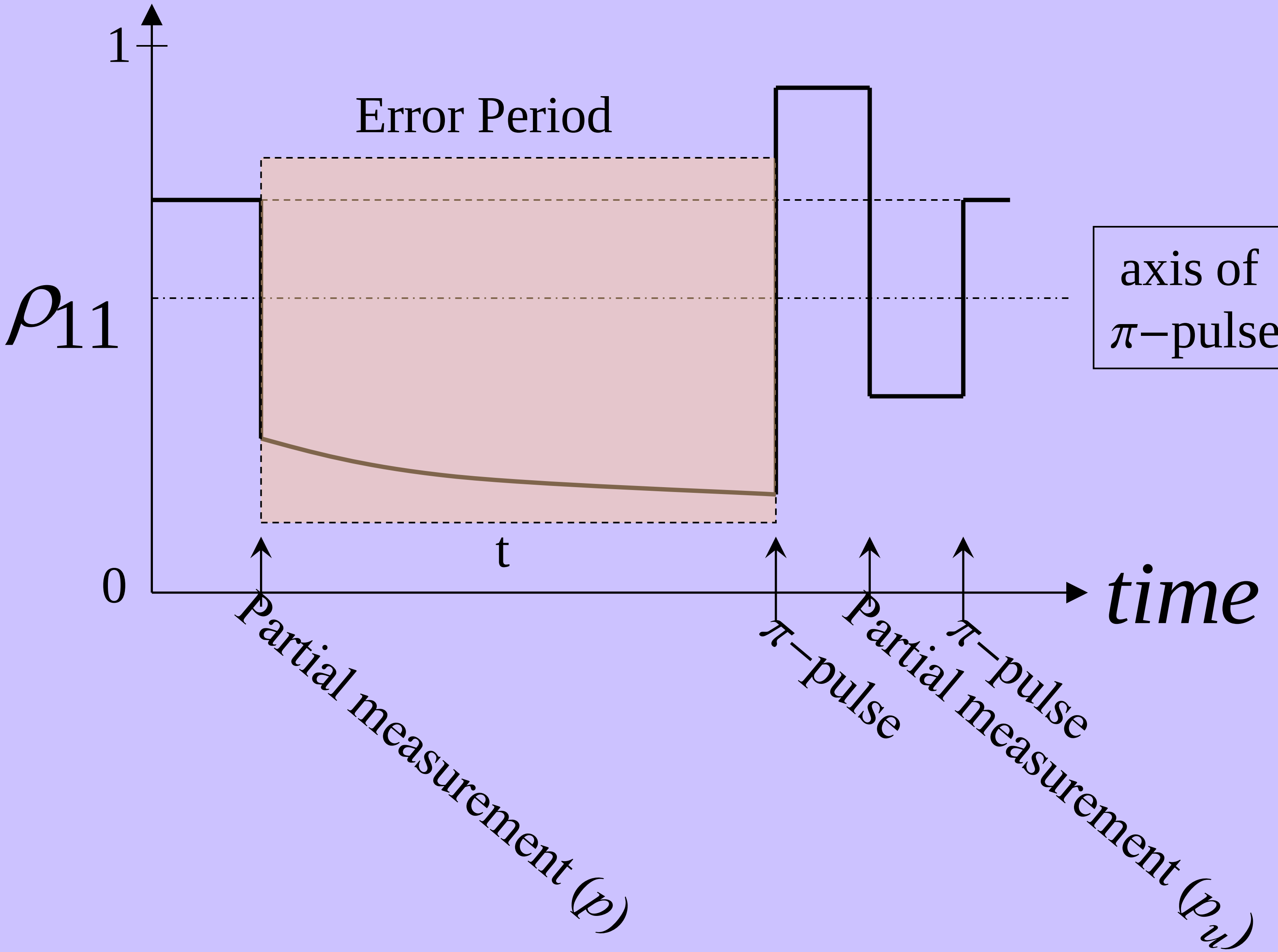
Fidelity of Numerical Results and Experiment



Status of Project 1
- Still in progress
- Main features of experimental results explained
- Investigating more sophisticated models
- Paper will be written soon
- Inspired project 2

N. Katz et al., Phys. Rev. Lett. 101, 200401 (2008)

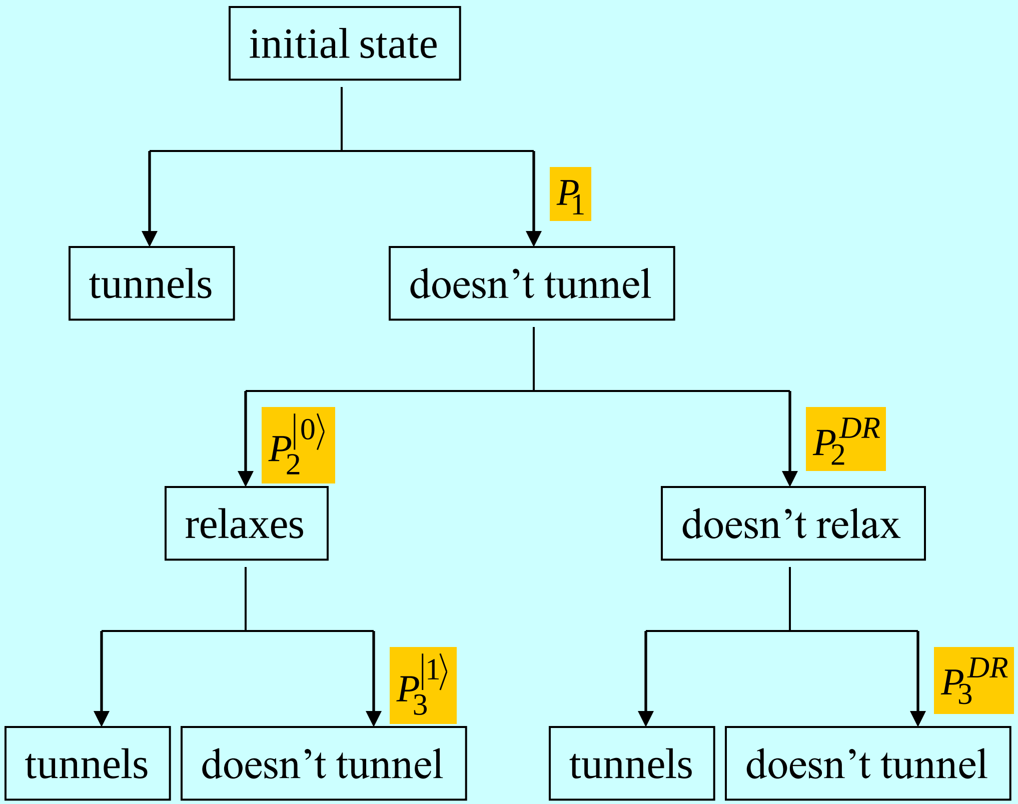
Project 2-Decoherence suppression using uncollapsing



Alexander N. Korotkov and Kyle Keane., Phys. Rev. A 81, 040103(R) (2010)

Why and How it should work

Ideal Operations

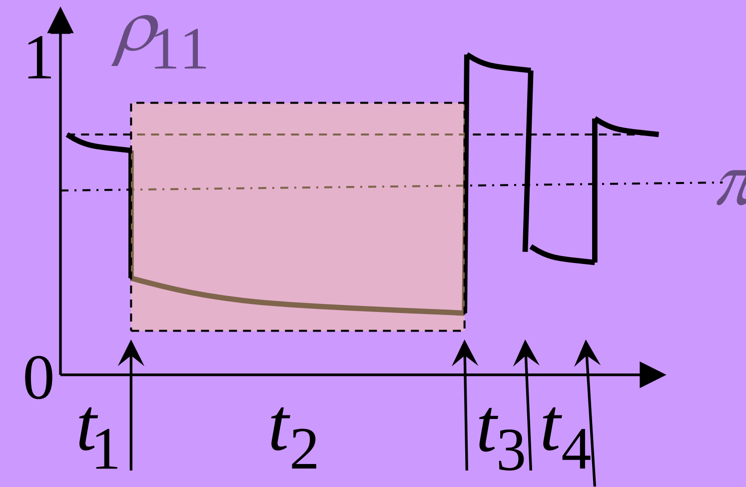


$$P_f^{|1\rangle} = P_1 P_2^{|0\rangle} P_3^{|1\rangle} = |\beta|^2 (1-p)^2 \left(1-e^{-t/T_1}\right) e^{-t/T_1}$$

$$P_f^G = P_1 P_2^{DR} P_3^{DR} = (1-p) e^{-t/T_1}$$

$$p_u = 1-(1-p) e^{-t/T_1} \qquad P^S = P_f^G + P_f^{|1\rangle}$$

Non-Ideal Operations

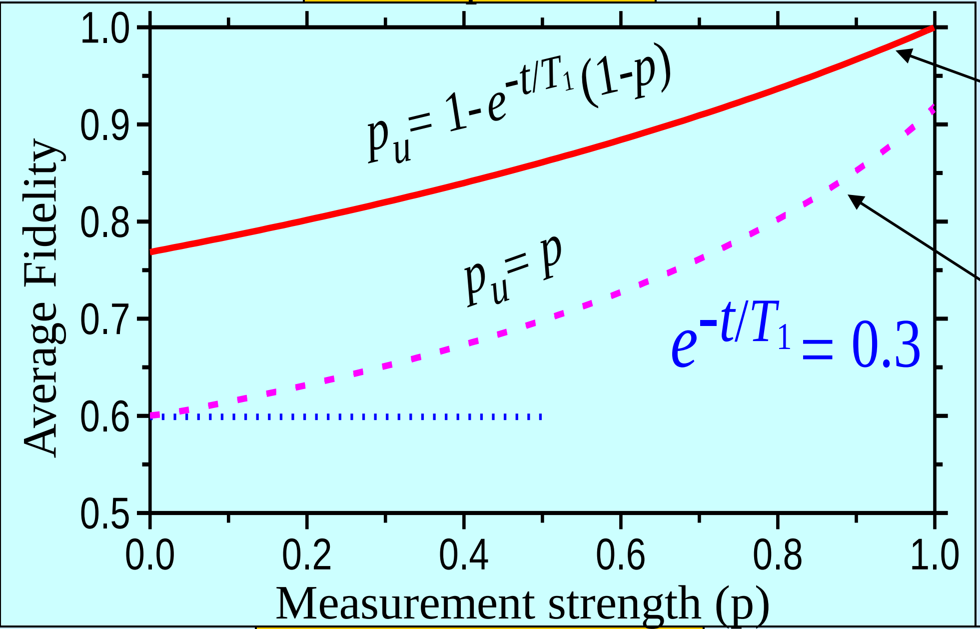


$$\kappa_m \equiv e^{-t_m/T_1}$$

$$\kappa_\phi \equiv e^{-\sum_m t_m/T_\phi}$$

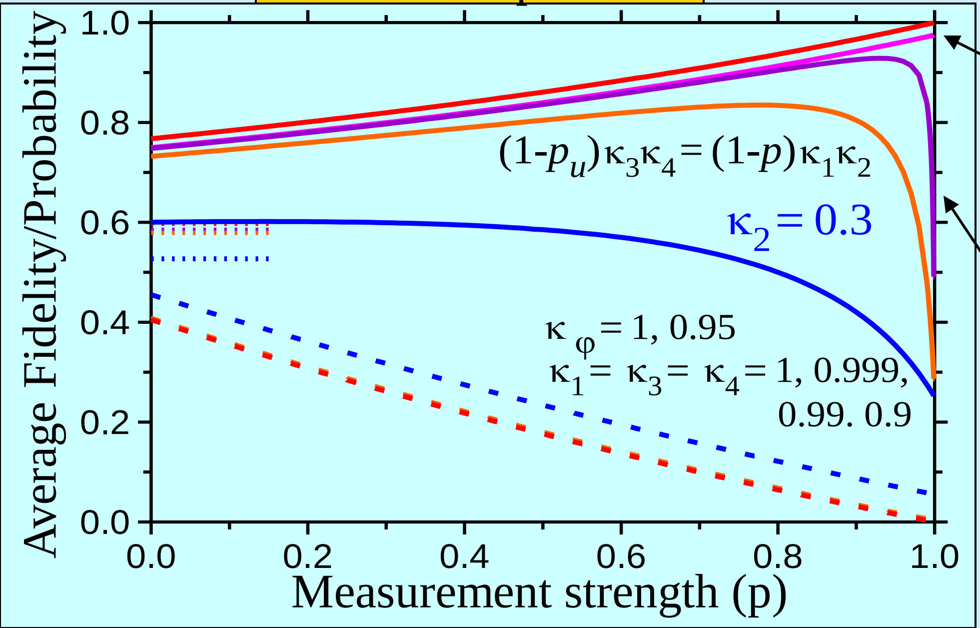
Results

Ideal Operations



- -Wise choice of uncollapsing measurement strength will return a state that is arbitrarily close to the initial state
- - Even a bad choice of uncollapsing strength will yield an improvement over pure relaxaed state

Non-Ideal Operations



- -Ideal operations with relaxation and dephasing during the error period, the ideally returned state is only slightly degraded
- - Improvement is still realizable in the presence of considerable decoherence during the operations, although perfect restoration is no longer achievement

K and K., Phys. Rev. A 81, 040103(R) (2010)

Project 3: Quantum Coding with Phase Qubits

Single Qubit Operations

Rotations

$$r^m(\theta) \equiv e^{-i\frac{\theta}{2}\sigma_m}$$

Single Qubit Rotations
In multiple qubit space

$$r_1^m(\theta) \equiv r^m(\theta) \otimes I$$

$$r_2^m(\theta) \equiv I \otimes r^m(\theta)$$

Mulitple Qubit Operation

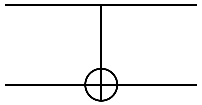
Cnot Gate

$$|00\rangle \rightarrow |00\rangle$$

$$|01\rangle \rightarrow |01\rangle$$

$$|10\rangle \rightarrow |11\rangle$$

$$|11\rangle \rightarrow |10\rangle$$



Classical Redundant Coding

Measure state of bit (no loss of superposition)

$$\boxed{\{\psi\} \to \{0\}} \to \{000\} \to \left[\{100\},\{010\},\{001\}\right] \to \{000\}$$

Create three copies

$$\boxed{\{0\} \to \{000\}} \to \left[\{100\},\{010\},\{001\}\right] \to \{000\}$$

Single bit flip error

$$\{0\} \to \boxed{\{000\} \to \left[\{100\},\{010\},\{001\}\right]} \to \{000\}$$

Measure all three bits and put all three in majority state

$$\{0\} \to \{000\} \to \boxed{\left[\{100\},\{010\},\{001\}\right] \to \{000\}}$$

Quantum Redundant Coding

~~Measure state of qubit~~

$$|\psi\rangle \rightarrow |\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle$$

Cannot create a copy by a unitary transformation!
No Cloning theorem

Cannot measure the superposition!
Projection onto Eigenvalue

What can we do? Entanglement.

Product State

$$\left(\alpha|0\rangle + \beta|1\rangle\right) \otimes |0\rangle \otimes |0\rangle = \alpha|000\rangle + \beta|100\rangle$$

Two CNOT gates

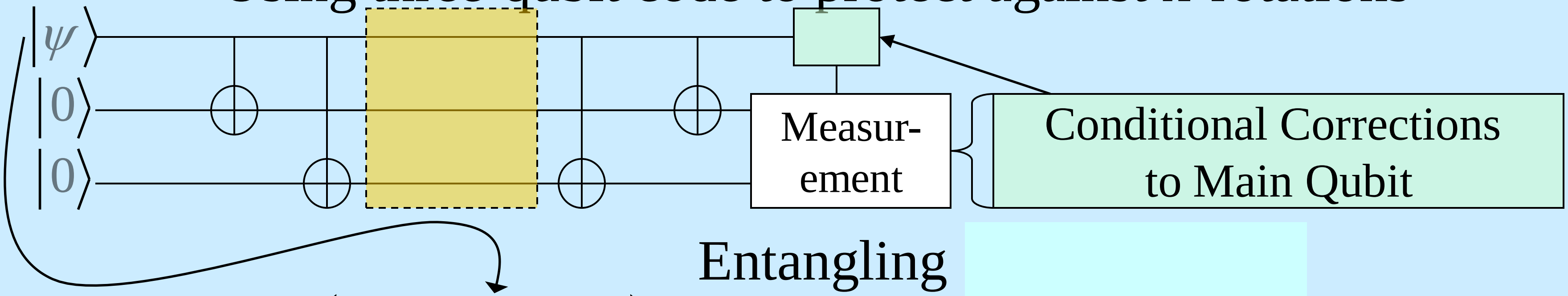
$$\alpha|000\rangle + \beta|100\rangle \rightarrow \underbrace{\alpha|000\rangle + \beta|111\rangle}$$

no longer product state
It is entangled!

That was fun, now what?

We have, in fact, entangled our system in such a way that x rotations of a single qubit can be detected and uniquely corrected!

Using three-qubit code to protect against x-rotations



Entangling

$$\left(\alpha|0\rangle+\beta|1\rangle\right)\otimes|0\rangle|0\rangle \rightarrow \alpha|000\rangle+\beta|111\rangle$$

X rotation of first qubit → $r_1^x(2\theta)$

$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|100\rangle - i\beta\sin(\theta)|011\rangle + \beta\cos(\theta)|111\rangle$$

Un-entangling

$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|111\rangle - i\beta\sin(\theta)|011\rangle + \beta\cos(\theta)|100\rangle$$

Rearrangement of terms

Bit Flip Needed / *Bit Flip Needed*

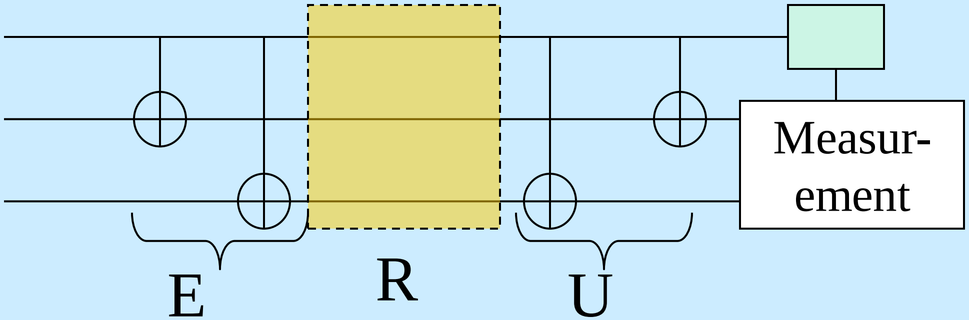
$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\beta|0\rangle+\alpha|1\rangle\right)|11\rangle \quad r_1^x(2\theta)$$

$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|10\rangle \quad r_2^x(2\theta)$$

$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|01\rangle \quad r_3^x(2\theta)$$

No Correction Needed / *No Correction Needed*

Can the three-qubit code protect against relaxation?



Relaxation seems to be similar to a bit flip in that it takes $|1\rangle \rightarrow |0\rangle$

At finite temperature there are also excitations that take $|0\rangle \rightarrow |1\rangle$

Aren't these like a bit flip?

First qubit relaxes

$$\left(\alpha|0\rangle + \beta|1\rangle\right) \otimes |0\rangle|0\rangle \xrightarrow{E} \alpha|000\rangle + \beta|111\rangle \xrightarrow{R} |0\rangle|11\rangle \xrightarrow{U} |0\rangle|00\rangle$$

$|0\rangle \otimes |11\rangle$ 1^{st} qubit relaxes

Cannot be restored

Similar for other two qubits

$|1\rangle \otimes |10\rangle$ 2^{nd} qubit relaxes

$|1\rangle \otimes |01\rangle$ 3^{rd} qubit relaxes

Cannot be restored

Energy Relaxation

Relaxation can be represented as
a **projective measurement** onto |1> followed by a Pi rotation

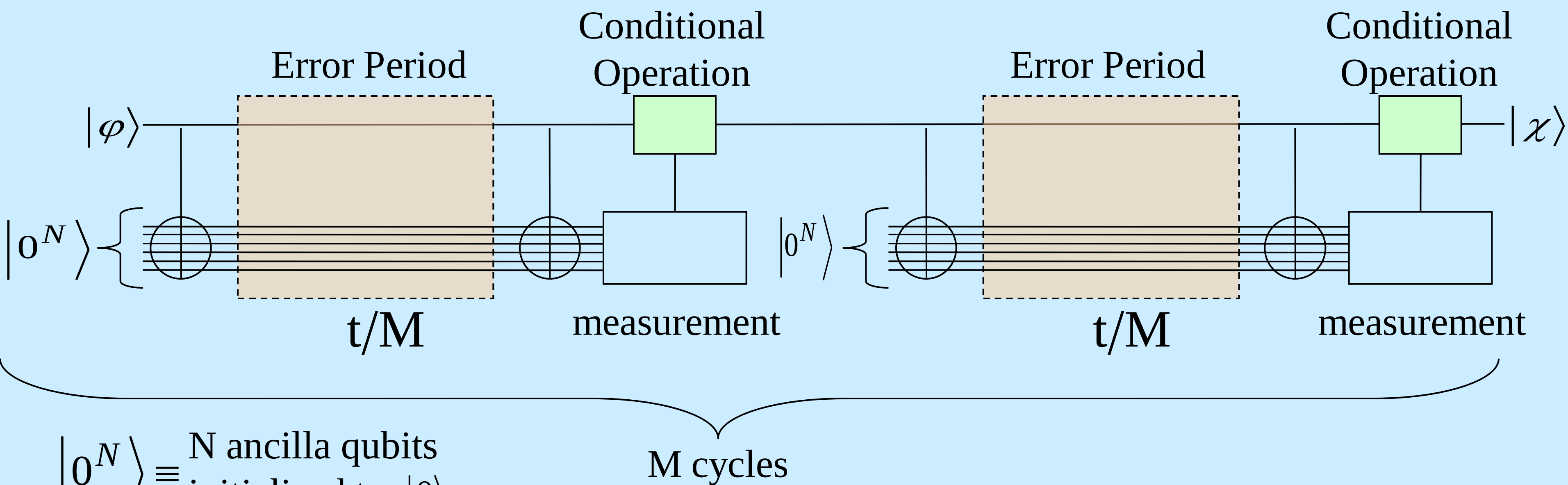
$$\alpha\left|0\right\rangle + \beta\left|1\right\rangle \rightarrow \left|1\right\rangle \rightarrow \left|0\right\rangle$$

The trick of quantum error correction is to
indirectly measure what has happened to the qubit and correct the dynamic change.
Do this without extracting any information about the state of the qubit.
Any extracted information changes the qubit state.

In the 5 (or 7 or 9) qubit code, a projective measurement will yield no information about the original superposition and therefore leaves the original superposition unchanged, and also therefore protects against energy relaxation

Project 3b-Performance of detection codes with relaxation

N qubit, M cycles



$|0^N\rangle \equiv$ N ancilla qubits initialized to $|0\rangle$

Although these codes will not correct for relaxation

-Can they be used to detect and discard relaxation errors?

-Will adding more ancilla qubits improve the performance?

-Can we repeat this fast enough to suppress the chance of having an error?

-Can we repeat this fast enough to have perfect fidelity of the retained qubit?

Analytics of Fianl State

CNOTs to all ancilla qubits

$$\alpha\left|0^N\right\rangle+\beta\left|1\left\{0^{N-1}\right\}\right\rangle \rightarrow \alpha\left|0^N\right\rangle+\beta\left|1\left\{1^{N-1}\right\}\right\rangle$$

No qubits relax

No qubits relax qubit ends in $\left|\psi\right\rangle$	$\alpha\left|0^N\right\rangle+\beta\left|1^N\right\rangle \rightarrow \left|\psi\right\rangle \equiv \frac{\alpha\left|0^N\right\rangle+\beta(1-p)^{N/2}\left|1^N\right\rangle}{\sqrt{\|\alpha\|^2+\|\beta\|^2(1-p)^N}}$ with prob $\|\alpha\|^2+\|\beta\|^2(1-p)^N$

One qubit relaxes

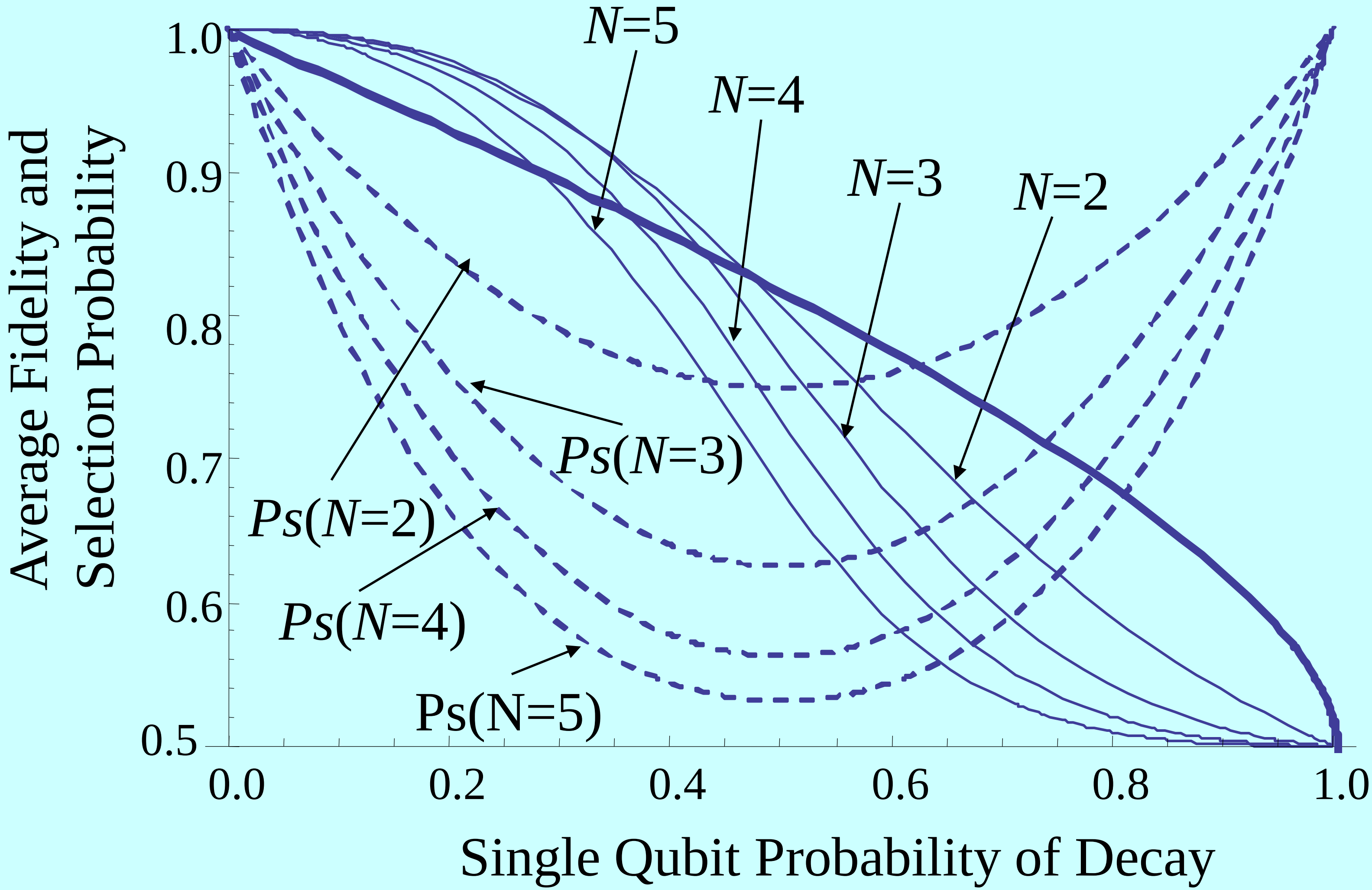
First qubit relaxes qubit ends in $\left\|0\right\rangle$	$\alpha\left\|0^N\right\rangle+\beta\left\|1^N\right\rangle \rightarrow \left\|0,\left\{1^N\right\}\right\rangle$ with prob $\binom{\text{N-1}}{0}\|\beta\|^2 p(1-p)^{N-1}$
Ancilla qubit relaxes qubit ends in $\left\|1\right\rangle$	$\alpha\left\|0^N\right\rangle+\beta\left\|1^N\right\rangle \rightarrow \left\|1,\left\{0,1^{N-1}\right\}\right\rangle$ with prob $\binom{\text{N-1}}{1}\|\beta\|^2 p(1-p)^{N-1}$

Two qubits relax

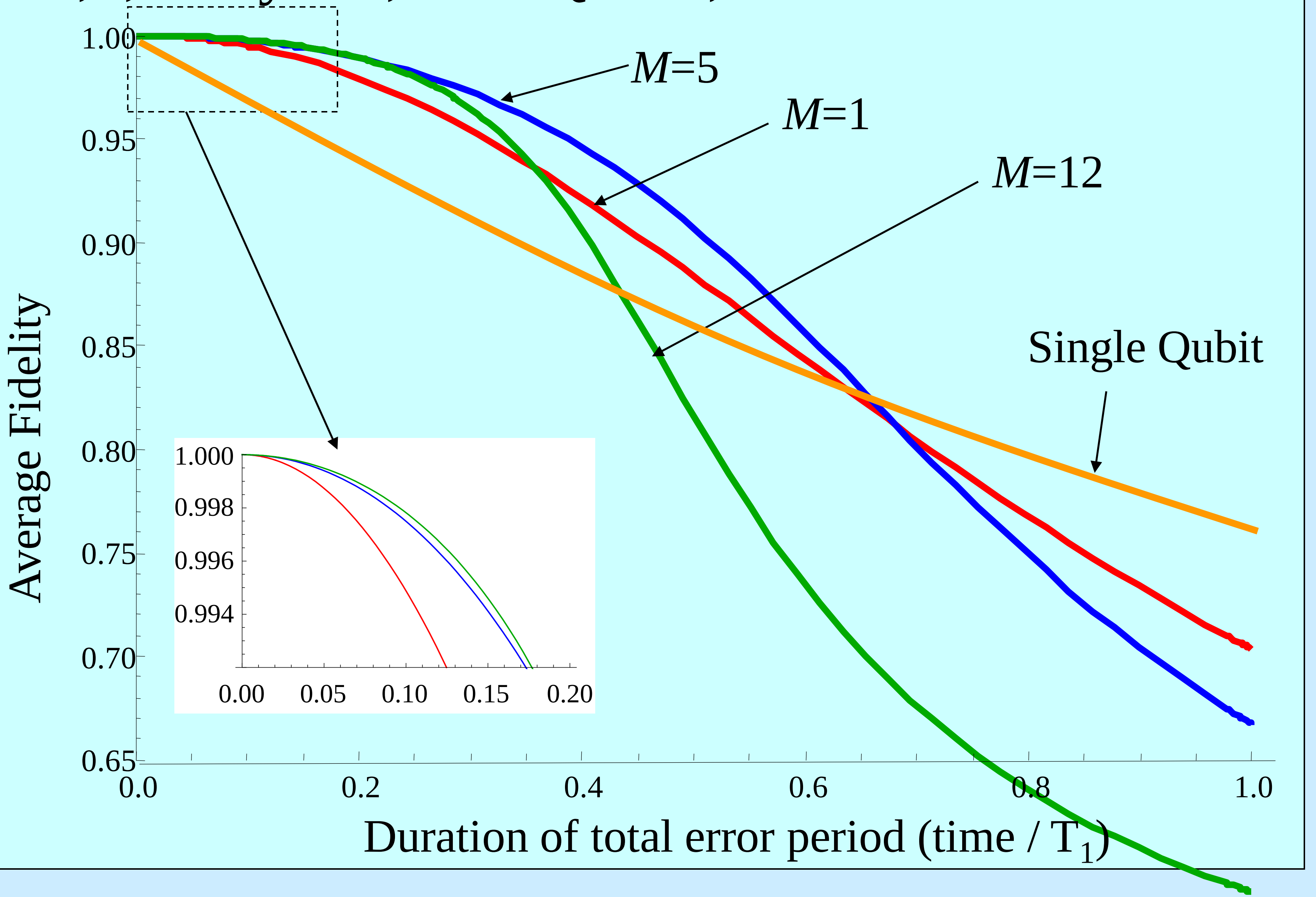
First qubit relaxes qubit ends in $\left\|0\right\rangle$	$\alpha\left\|0^N\right\rangle+\beta\left\|1^N\right\rangle \rightarrow \left\|0,\left\{0,1^{N-1}\right\}\right\rangle$ with prob $\binom{\text{N-1}}{1}\|\beta\|^2 p^2(1-p)^{N-2}$
Ancilla qubit relaxes qubit ends in $\left\|1\right\rangle$	$\alpha\left\|0^N\right\rangle+\beta\left\|1^N\right\rangle \rightarrow \left\|1,\left\{0,0,1^{N-2}\right\}\right\rangle$ with prob $\binom{\text{N-1}}{2}\|\beta\|^2 p^2(1-p)^{N-2}$

$$F_{state} = P_{|\psi\rangle}F_{|\psi\rangle} + P_{|0\rangle}F_{|0\rangle} + P_{|1\rangle}F_{|1\rangle}$$

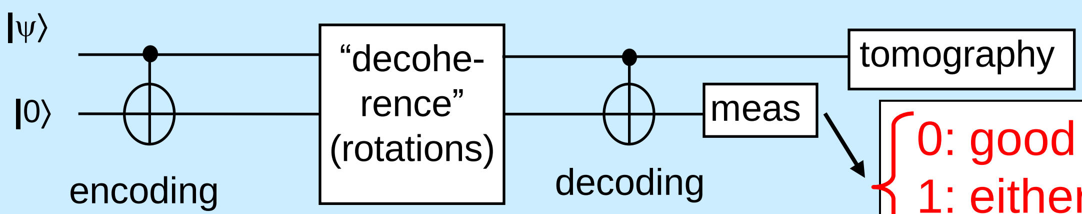
One Cycle, *N* Qubits, Selection of Result 0



1,5,12 Cycles, Two Qubits, Selection of Result 0



Project 3c-Two Qubit Quantum Error Detection of Rotations



0: good
1: either discard (only detection)
or correct (if know which error)

X-rotation of first qubit by **arbitrary** angle 2φ:

$$\alpha|00\rangle+\beta|11\rangle \to \cos\varphi(\alpha|00\rangle+\beta|11\rangle)-i\sin\varphi(\alpha|10\rangle+\beta|01\rangle)\to$$
$$\to \cos\varphi(\alpha|00\rangle+\beta|10\rangle)-i\sin\varphi(\alpha|11\rangle+\beta|01\rangle)=$$
$$=\cos\varphi\boxed{(\alpha|0\rangle+\beta|1\rangle)|0\rangle}-i\sin\varphi\underbrace{(\alpha|1\rangle+\beta|0\rangle)}_{X\text{-correction needed}}|1\rangle$$

X-rotation of second qubit by angle 2φ:

$$\alpha|00\rangle+\beta|11\rangle \to \cos\varphi(\alpha|00\rangle+\beta|11\rangle)-i\sin\varphi(\alpha|01\rangle+\beta|10\rangle)\to$$
$$\to \cos\varphi\boxed{(\alpha|0\rangle+\beta|1\rangle)|0\rangle}-i\sin\varphi\underbrace{(\alpha|0\rangle+\beta|1\rangle)}_{\text{no correction needed}}|1\rangle$$

Now *Y*-rotation of first qubit by angle 2φ:

$$\alpha|00\rangle+\beta|11\rangle \to \cos\varphi(\alpha|00\rangle+\beta|11\rangle)+\sin\varphi(\alpha|10\rangle-\beta|01\rangle)\to$$
$$\to \cos\varphi\boxed{(\alpha|0\rangle+\beta|1\rangle)|0\rangle}+\sin\varphi\underbrace{(\alpha|1\rangle-\beta|0\rangle)}_{\text{needs } Y}|1\rangle$$

Y-rotation of second qubit by angle 2φ:

$$\alpha|00\rangle+\beta|11\rangle \to \cos\varphi(\alpha|00\rangle+\beta|11\rangle)+\sin\varphi(\alpha|01\rangle-\beta|10\rangle)\to$$
$$\to \cos\varphi\boxed{(\alpha|0\rangle+\beta|1\rangle)|0\rangle}+\sin\varphi\underbrace{(\alpha|0\rangle-\beta|1\rangle)}_{\text{needs Z}}|1\rangle$$

Dashed lines are the probability of ancilla result 1

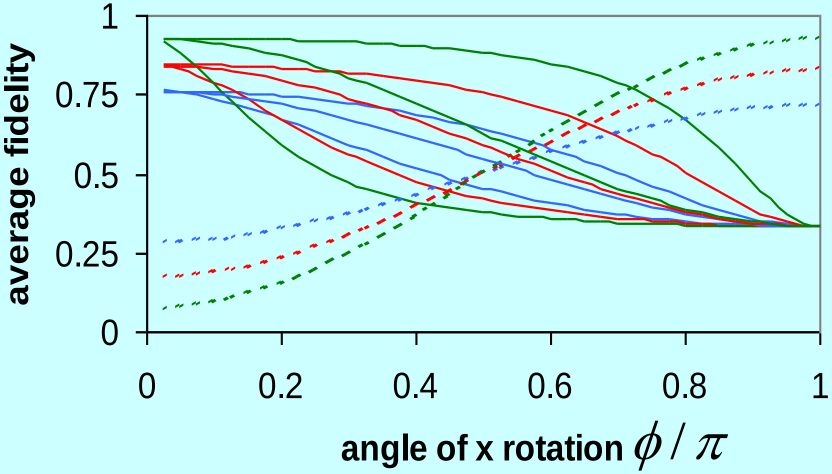
Performance with Pure Dephasing

Duration = 130ns

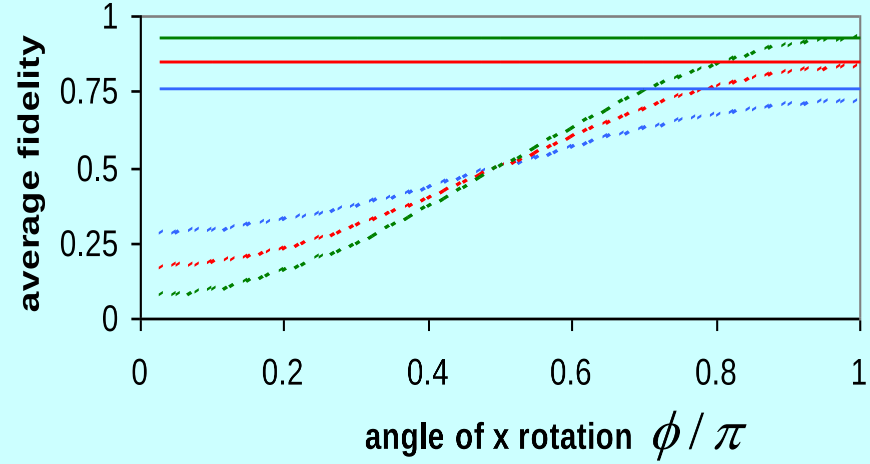
$T_2 = 650ns$ $T_2 = 250ns$ $T_2 = 125ns$

$T_1 = \infty$

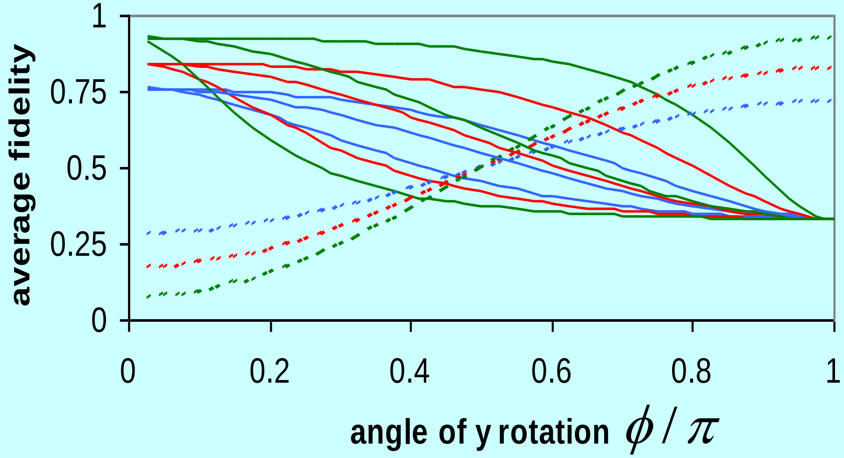
x rotation of main qubit



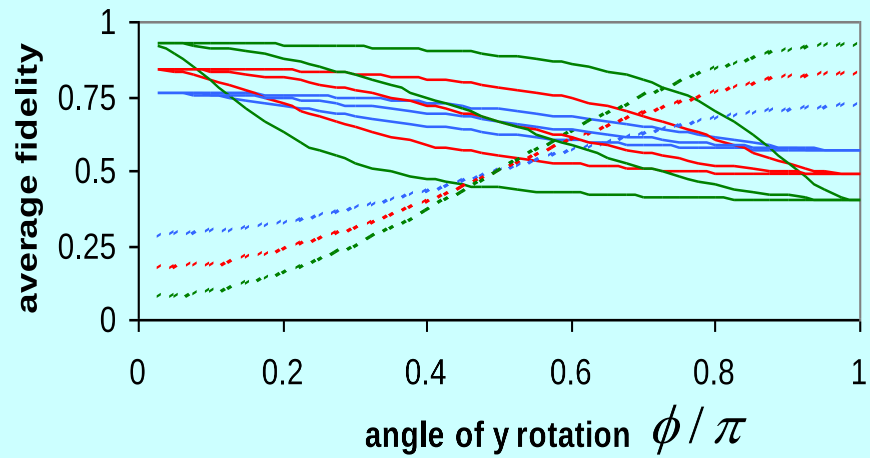
x rotation of ancilla qubit



y rotation of main qubit



y rotation of ancilla qubit



Future directions

Understanding performance of codes in presence
of multiple errors and optimizing
experimental visibility and implementation

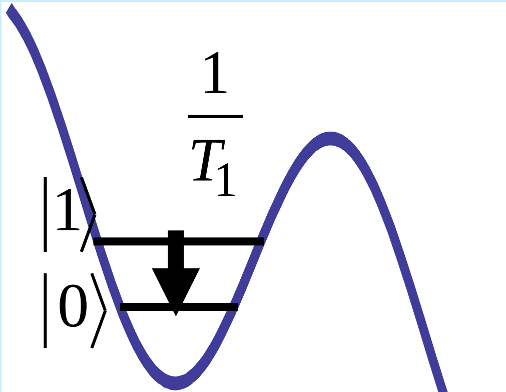
Quantum process tomography: how to extract specific information about
a process from the Chi matrix

Theoretical support of experimental progress

Appendices

Representations of Errors-Example: Energy Relaxation Master Equation

RETURN



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$p \equiv 1 - e^{-t/T_1}$$

$$\frac{\partial}{\partial t}\rho_{11}(t) = -\frac{1}{T_1}\rho_{11}(t)$$

<u>Need to derive this from commutator!!!!!</u>

$$\frac{\partial}{\partial t}\rho_{00}(t) = -\frac{\partial}{\partial t}\rho_{11}(t)$$

From the normalization requirement

$$\frac{\partial}{\partial t}\rho_{01}(t) = -\frac{1}{2T_1}\rho_{01}(t)$$

$$\frac{\partial}{\partial t}\rho_{10}(t) = -\frac{1}{2T_1}\rho_{10}(t)$$

<u>Need to derive this from somewhere!!!!!</u>

Solving these equations and combining into an operation

$$D[\rho] \to \begin{pmatrix} 1-\rho_{11}(1-p) & \rho_{01}\sqrt{1-p} \\ \rho_{10}\sqrt{1-p} & \rho_{11}(1-p) \end{pmatrix}$$

Choosing a specific operator sum decomposition

$$D[\rho] = K_R \rho K_R^{\dagger} + K_{DR}\rho K_{DR}^{\dagger} \qquad K_R = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix} \qquad K_{DR} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

If you initially have a pure state, the classical mixture created by this process becomes explicit

$$\rho(t) = K_R|\psi\rangle\langle\psi|K_R^{\dagger} + K_{DR}|\psi\rangle\langle\psi|K_{DR}^{\dagger} = P_R|\psi_R\rangle\langle\psi_R| + P_{DR}|\psi_{DR}\rangle\langle\psi_{DR}|$$

LINK

$$|\psi\rangle \to \begin{cases} \dfrac{\alpha|0\rangle + \beta\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}, \text{ with probability } |\alpha|^2 + |\beta|^2(1-p) \\ |0\rangle, \text{ with probability } |\beta|^2\, p \end{cases}$$

This can be done for any operation however only some give physically meaningful interpretations

RETURN

Representation of experiment specific errors

Pure Dephasing

$$PD[\rho] \rightarrow \begin{pmatrix} \rho_{00} & \rho_{01}\, e^{-t/T_\phi} \\ \rho_{10}\, e^{-t/T_\phi} & \rho_{11} \end{pmatrix}$$

$$K_{DT} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \quad K_{DT} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

$$\frac{\partial}{\partial t}\rho_{11}(t) = \frac{\partial}{\partial t}\rho_{00}(t) = 0$$

$$\frac{\partial}{\partial t}\rho_{01}(t) = -\frac{1}{T_\phi}\rho_{01}(t)$$

$$\frac{\partial}{\partial t}\rho_{10}(t) = -\frac{1}{T_\phi}\rho_{10}(t)$$

$$d \equiv 1 - e^{-t/T_\phi}$$

Partial Measurement

$$\frac{\partial}{\partial t}\rho_{11}(t) = -\Gamma \rho_{11}(t)$$

$$\frac{\partial}{\partial t}\rho_{00}(t) = 0$$

$$\frac{\partial}{\partial t}\rho_{01}(t) = -\frac{\Gamma}{2}\rho_{01}(t)$$

$$\frac{\partial}{\partial t}\rho_{10}(t) = -\frac{\Gamma}{2}\rho_{10}(t)$$

$$PM[\rho] \rightarrow \frac{1}{\rho_{00} + \rho_{11}(1-p)} \begin{pmatrix} \rho_{00} & \rho_{01}\sqrt{1-p} \\ \rho_{10}\sqrt{1-p} & \rho_{11}(1-p) \end{pmatrix}$$

$$K_{PM} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

Probabilities for Decoherence Suppression

$$\alpha|0\rangle+\beta|1\rangle \xrightarrow{P_1} \frac{\alpha|0\rangle+\beta\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \xrightarrow{P_2^{DR}} \frac{\alpha|0\rangle+\beta\sqrt{1-p}e^{-t/2T_1}|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)e^{-t/T_1}}} \xrightarrow{1} \frac{\beta\sqrt{1-p}e^{-t/2T_1}|0\rangle+\alpha|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)e^{-t/T_1}}}$$

$$\frac{\alpha|0\rangle+\beta\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}} \xrightarrow{P_2^{|0\rangle}} |0\rangle \xrightarrow{1} |1\rangle$$

$$\xrightarrow{P_3^{DR}} \frac{\beta\sqrt{1-p}e^{-t/2T_1}|0\rangle+\alpha\sqrt{1-p_u}\,|1\rangle}{\sqrt{|\alpha|^2(1-p_u)+|\beta|^2(1-p)e^{-t/T_1}}} = \beta|0\rangle+\alpha|1\rangle$$

$$\xrightarrow{P_3^{|1\rangle}} |1\rangle$$

$$p_u = 1-(1-p)e^{-t/T_1}$$

$$P_1 = |\alpha|^2+|\beta|^2(1-p)$$

$$P_2^{DR} = \frac{|\alpha|^2+|\beta|^2(1-p)e^{-t/T_1}}{|\alpha|^2+|\beta|^2(1-p)}$$

$$P_2^{|0\rangle} = \frac{|\beta|^2(1-p)\left(1-e^{-t/T_1}\right)}{|\alpha|^2+|\beta|^2(1-p)}$$

$$P_3^{DR} = \frac{|\alpha|^2(1-p_u)+|\beta|^2(1-p)e^{-t/T_1}}{|\alpha|^2+|\beta|^2(1-p)e^{-t/T_1}}$$

$$P_3^{|1\rangle} = (1-p_u)$$

$$P_f^{|1\rangle} = P_1P_2^{|0\rangle}P_3^{|1\rangle} = |\beta|^2(1-p)^2\left(1-e^{-t/T_1}\right)e^{-t/T_1}$$

$$P_f^{G} = P_1P_2^{DR}P_3^{DR} = (1-p)e^{-t/T_1}$$

Josephson Junction-Phase Dynamics

$$I_J = I_0 \sin(\delta)$$

$$\delta = \frac{2\pi}{\Phi_0}\int V_J\,dt = \frac{2\pi}{\Phi_0}\Phi_\delta$$

$$V_J = \frac{\Phi_0}{2\pi}\frac{\partial\delta}{\partial t}$$

$$\frac{\partial I_J}{\partial t} = I_0\cos(\delta)\frac{2\pi}{\Phi_0}V_J$$

$$L_J = \left(I_0\cos(\delta)\frac{2\pi}{\Phi_0}\right)^{-1}$$

I_J is the supercurrent through junction

I_0 is the critical current of the junction

δ is the phase difference across the junction

$\Phi_0 = \frac{h}{2e}$ is the superconducting flux quantum

V_J is the voltage across the junction

$\phi = \frac{\Phi_{ext}}{\Phi_0}$ is the number of flux quanta applied

$$E_C = \frac{1}{2}CV^2$$

$$E_L = \frac{1}{2}LI^2$$

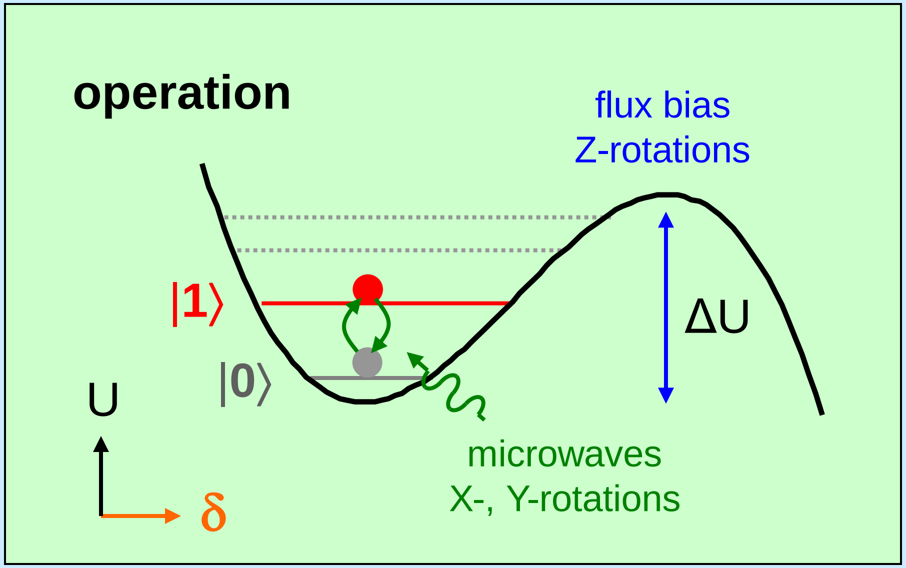
$$U_\delta = \int I_J V_J\,dt = -\frac{I_0\Phi_0}{2\pi}\cos(\delta)$$

$$U_\Phi = \frac{1}{2}LI^2 = \frac{1}{2L}\Phi_t^2 = \frac{1}{2L}\left(\Phi_{ext} - \Phi_\delta\right)^2$$

$$U_\Phi = \frac{1}{2L}\left(\frac{\Phi}{\Phi_0} - \frac{\Phi_0\,\delta}{2\pi}\right)^2$$

$$U = U_\delta + U_\Phi$$

$$U = -\frac{I_0\Phi_0}{2\pi}\cos(\delta) + \frac{\Phi_0^2}{2L}\left(\phi - \frac{\delta}{2\pi}\right)^2$$



RETURN

Kraus Operator to Mixture of Pure States

$$K_R = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix} \qquad K_{DR} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad |\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

$$K_R|\psi\rangle\langle\psi|K_R^\dagger = \begin{pmatrix} |\beta|^2 p & 0 \\ 0 & 0 \end{pmatrix} \to |\beta|^2 p \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \to |\beta|^2 p|0\rangle\langle 0| \equiv P_R|\psi_R\rangle\langle\psi_R|$$

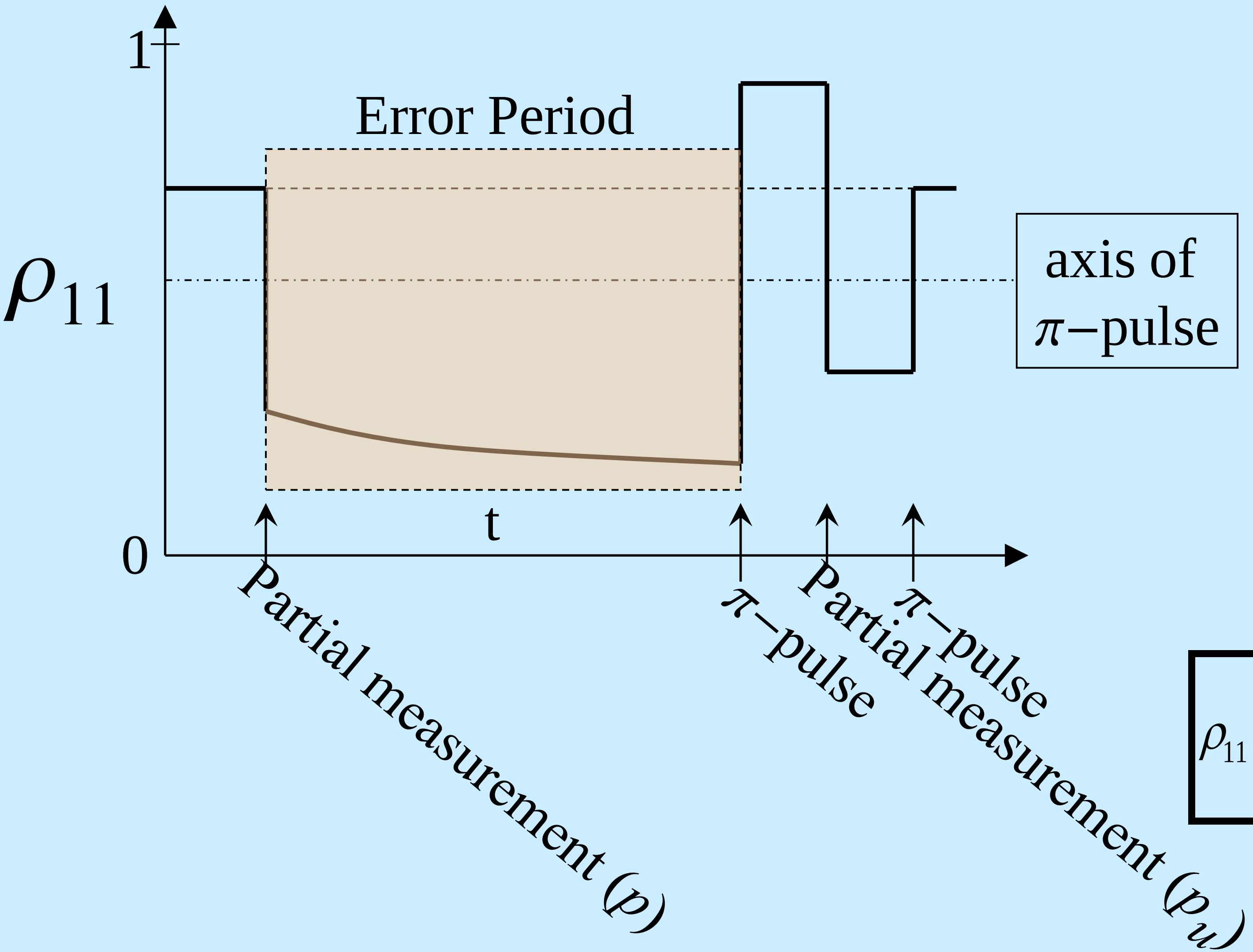
$$K_{DR}|\psi\rangle\langle\psi|K_{DR}^\dagger = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\sqrt{1-p} \\ \alpha^*\beta\sqrt{1-p} & |\beta|^2(1-p) \end{pmatrix} \to \left(|\alpha|^2 + |\beta|^2 p\right)\frac{1}{|\alpha|^2 + |\beta|^2 p}\begin{pmatrix} |\alpha|^2 & \alpha\beta^*\sqrt{1-p} \\ \alpha^*\beta\sqrt{1-p} & |\beta|^2(1-p) \end{pmatrix}$$

$$\to \left(|\alpha|^2 + |\beta|^2 p\right)\left(\frac{\alpha|0\rangle + \beta\sqrt{1-p}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}\right)\left(\frac{\alpha^*\langle 0| + \beta^*\sqrt{1-p}\langle 1|}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}\right) \equiv P_{DR}|\psi_{DR}\rangle\langle\psi_{DR}|$$

$$\rho(t) = K_R|\psi\rangle\langle\psi|K_R^\dagger + K_{DR}|\psi\rangle\langle\psi|K_{DR}^\dagger = P_R|\psi_R\rangle\langle\psi_R| + P_{DR}|\psi_{DR}\rangle\langle\psi_{DR}|$$

$$|\psi\rangle \to \begin{cases} \dfrac{\alpha|0\rangle + \beta\sqrt{1-p}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}, \text{with probability } |\alpha|^2 + |\beta|^2(1-p) \\ |0\rangle, \text{with probability } |\beta|^2 p \end{cases}$$

Project 2-Decoherence suppression using uncollapsing



Partial Measurement

$$\psi_0 = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$\psi_{pm1} = \frac{\alpha}{N}\,|0\rangle + \frac{\beta\sqrt{1-p}}{N}\,|1\rangle$$

$$N \equiv \sqrt{|\alpha|^2 + |\beta|^2\,(1-p)}$$

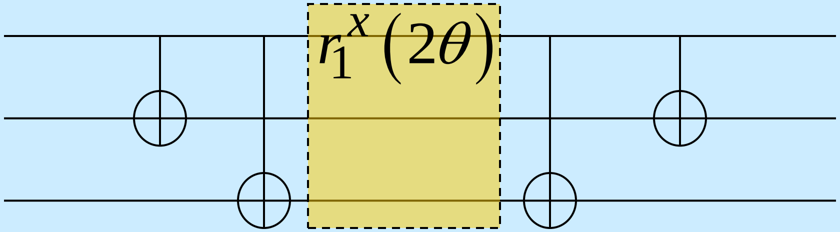
$$\frac{\alpha}{N} > \alpha \qquad \frac{\beta\sqrt{1-p}}{N} < \beta$$

$$\rho_{11} \to \rho_{11}\frac{(1-p)}{\rho_{00} + \rho_{11}(1-p)}$$

Markovian Relaxation

$$\rho_{11} \to \rho_{11} e^{-t/T_1}$$

Using three-qubit codes to protect



$$\left(\alpha|0\rangle+\beta|1\rangle\right)\otimes|0\rangle\otimes|0\rangle=\alpha|000\rangle+\beta|100\rangle$$

$$C_{12}C_{13}\left(\alpha|000\rangle+\beta|100\rangle\right)=\alpha|000\rangle+\beta|111\rangle$$

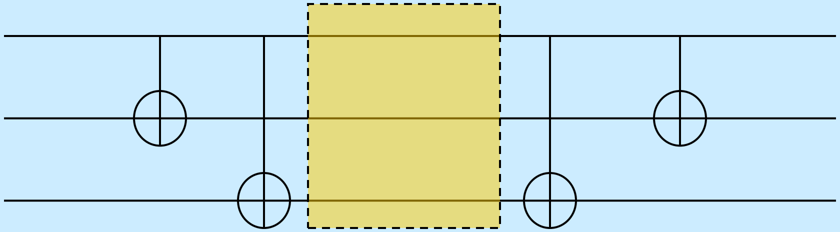
$$r_1^x(2\theta)\left(\alpha|000\rangle+\beta|111\rangle\right)=$$

$$\alpha\cos(\theta)|000\rangle-i\alpha\sin(\theta)|100\rangle-i\beta\sin(\theta)|011\rangle+\beta\cos(\theta)|111\rangle$$

$$C_{13}C_{12}r_1^x(2\theta)\left(\alpha|000\rangle+\beta|111\rangle\right)=$$

$$\alpha\cos(\theta)|000\rangle-i\alpha\sin(\theta)|111\rangle-i\beta\sin(\theta)|011\rangle+\beta\cos(\theta)|100\rangle$$

$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle-i\sin(\theta)\left(\beta|0\rangle+\alpha|1\rangle\right)|10\rangle$$



Using three-qubit code to protect against relaxation

$$\alpha|000\rangle + \beta|100\rangle \to \alpha|000\rangle + \beta|111\rangle \to \left(\frac{\alpha|0\rangle + \beta\sqrt{1-p}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)}}\right)|00\rangle$$

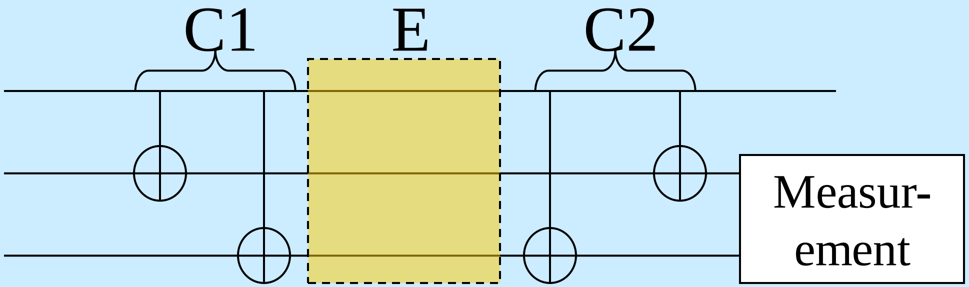
$$|011\rangle$$
$$|101\rangle$$
$$|110\rangle$$

$$|011\rangle$$
$$|110\rangle$$
$$|101\rangle$$

$|0\rangle \otimes |11\rangle$ — 1st qubit relaxes

$|1\rangle \otimes |10\rangle$ — 2nd qubit relaxes

$|1\rangle \otimes |01\rangle$ — 3rd qubit relaxes



Can the three-qubit code protect against relaxation?

Allowing the possibility of relaxation on the first qubit only

First qubit does not relax

$$\alpha|000\rangle+\beta|100\rangle \xrightarrow{C1} \alpha|000\rangle+\beta|111\rangle \xrightarrow{E} \frac{\alpha|000\rangle+\beta\sqrt{1-p}\,|111\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}}$$

$$\xrightarrow{C2}\left(\frac{\alpha|0\rangle+\beta\sqrt{1-p}\,|1\rangle}{\sqrt{|\alpha|^2+|\beta|^2(1-p)}}\right)|00\rangle$$

Cannot be restored

First qubit relaxes

$$\alpha|000\rangle+\beta|100\rangle \xrightarrow{C1} \alpha|000\rangle+\beta|111\rangle \xrightarrow{E} |0\rangle|11\rangle \xrightarrow{C2} |0\rangle|00\rangle$$

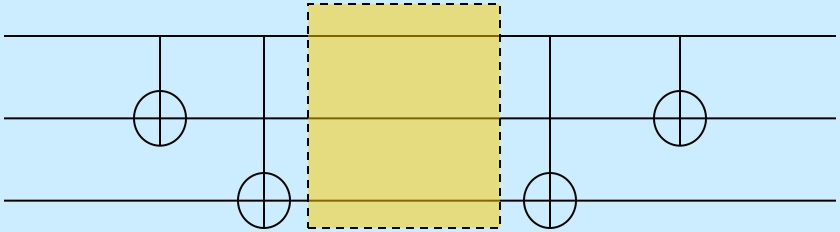
$|0\rangle\otimes|11\rangle$ — 1st qubit relaxes

Similarly

$|1\rangle\otimes|10\rangle$ — 2nd qubit relaxes

$|1\rangle\otimes|01\rangle$ — 3rd qubit relaxes

Using three-qubit code to protect against x-rotations



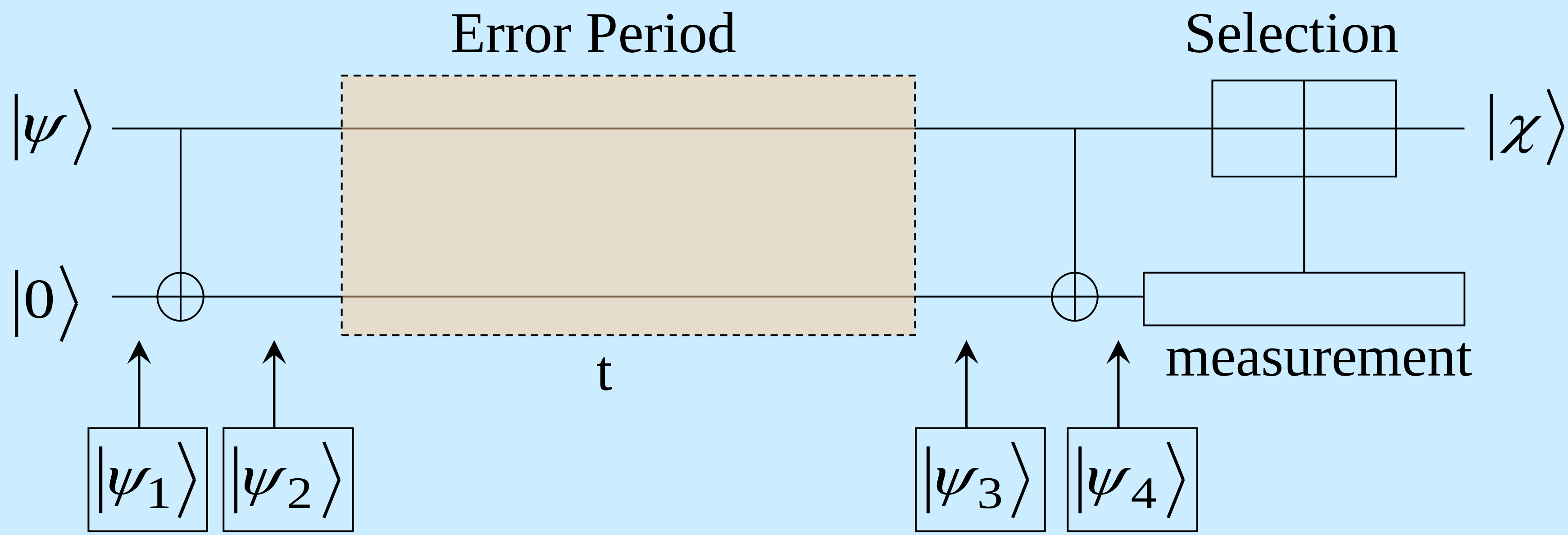
$$\alpha|000\rangle+\beta|100\rangle \rightarrow \alpha|000\rangle+\beta|111\rangle \rightarrow \left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle$$

$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|100\rangle - i\beta\sin(\theta)|011\rangle + \beta\cos(\theta)|111\rangle$$
$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|010\rangle - i\beta\sin(\theta)|101\rangle + \beta\cos(\theta)|111\rangle$$
$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|001\rangle - i\beta\sin(\theta)|110\rangle + \beta\cos(\theta)|111\rangle$$

$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|111\rangle - i\beta\sin(\theta)|011\rangle + \beta\cos(\theta)|100\rangle$$
$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|010\rangle - i\beta\sin(\theta)|110\rangle + \beta\cos(\theta)|100\rangle$$
$$\alpha\cos(\theta)|000\rangle - i\alpha\sin(\theta)|001\rangle - i\beta\sin(\theta)|101\rangle + \beta\cos(\theta)|100\rangle$$

$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\beta|0\rangle+\alpha|1\rangle\right)|11\rangle \qquad r_1^x(2\theta)$$
$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|10\rangle \qquad r_2^x(2\theta)$$
$$\cos(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|00\rangle - i\sin(\theta)\left(\alpha|0\rangle+\beta|1\rangle\right)|01\rangle \qquad r_3^x(2\theta)$$

Project 3c-Performance of two qubit detection code with relaxation



$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\psi_1\rangle = \alpha|00\rangle + \beta|10\rangle \qquad |\psi_2\rangle = \alpha|00\rangle + \beta|11\rangle$$

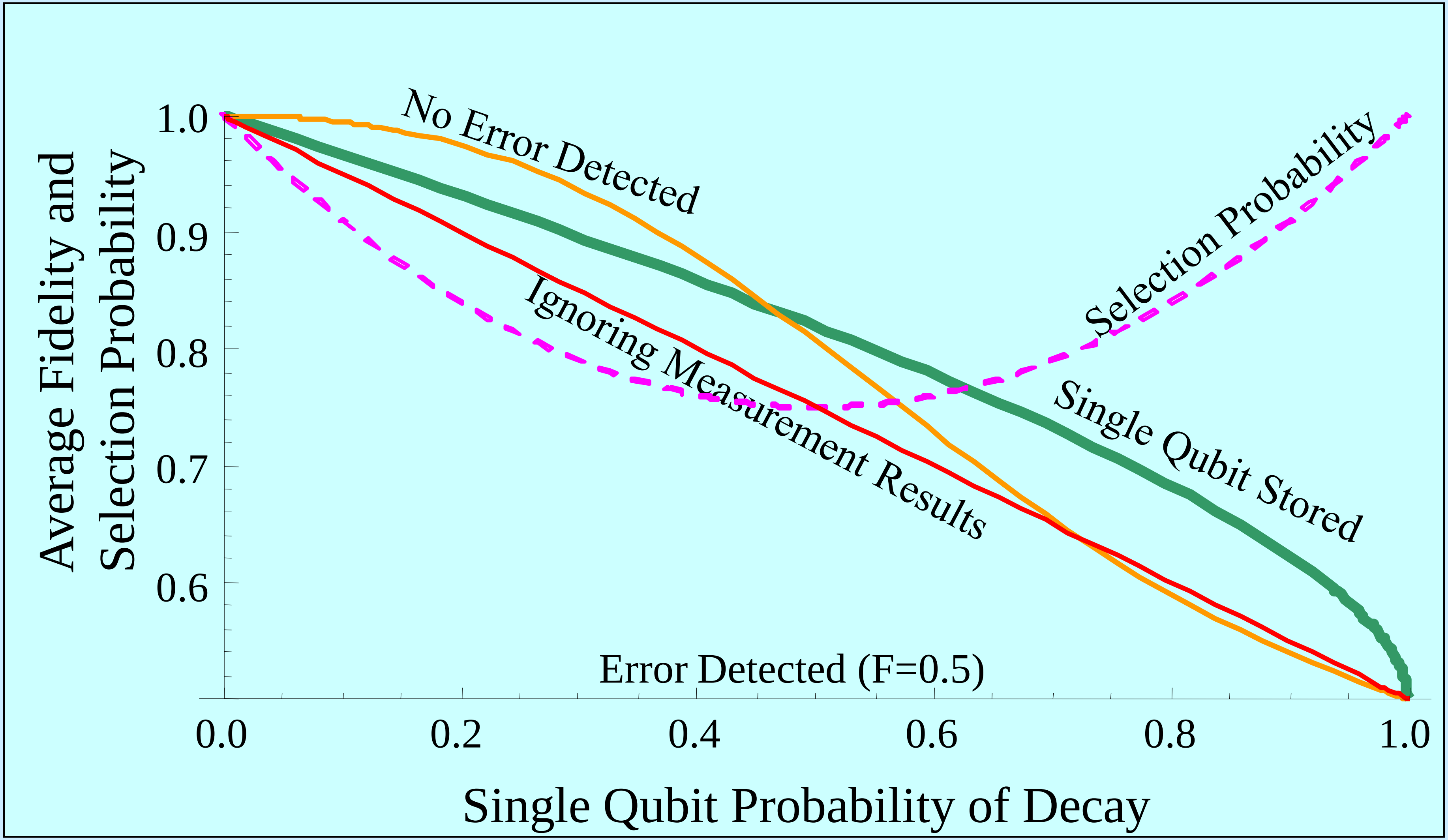
$$|\psi_3\rangle = \begin{cases} \dfrac{\alpha|00\rangle + \beta(1-p)|11\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)^2}}, \text{ with prob. } P_{nor} = |\alpha|^2 + |\beta|^2(1-p)^2 \\ |10\rangle, \text{ with probability } P_{10} = |\beta|^2(1-p)\,p \\ |01\rangle, \text{ with probability } P_{01} = |\beta|^2\, p(1-p) \\ |00\rangle, \text{ with probability } P_{00} = |\beta|^2\, p^2 \end{cases}$$

$$|\psi_4\rangle = \begin{cases} \dfrac{\alpha|0\rangle + \beta(1-p)|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2(1-p)^2}}|0\rangle, \text{ with prob. } P_{nor} = |\alpha|^2 + |\beta|^2(1-p)^2 \\ |1\rangle|1\rangle, \text{ with probability } P_{10} = |\beta|^2(1-p)\,p \\ |0\rangle|1\rangle, \text{ with probability } P_{01} = |\beta|^2\, p(1-p) \\ |0\rangle|0\rangle, \text{ with probability } P_{00} = |\beta|^2\, p^2 \end{cases}$$

$$\rho_0 = \frac{P_\chi|\chi\rangle\langle\chi| + P_{00}|0\rangle\langle 0|}{P_\chi + P_{00}}$$

$$\rho_1 = \frac{P_{10}|1\rangle\langle 1| + P_{01}|0\rangle\langle 0|}{P_{10} + P_{01}} \rightarrow \rho_1 = \frac{|1\rangle\langle 1| + |0\rangle\langle 0|}{2}$$

Performance



$$\left.\begin{array}{r}\frac{\partial}{\partial t}\rho_{00}(t)=0\rightarrow\rho_{00}(t)=\rho_{00}(0)\\ \frac{\partial}{\partial t}\rho_{11}(t)=-\Gamma\rho_{11}(t)\rightarrow\\ \rho_{11}(t)=\rho_{11}(0)e^{-\Gamma t}\equiv\rho_{11}(0)\left(1-p\left(\Gamma,t\right)\right)\end{array}\right\}A_{T^C}=\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-p_t(t)}\end{pmatrix}$$